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THE MONIST

ON THE FORM AND SPECTRUM OF ATOMS.*

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CHEMISTRY teaches that all earthly matter is composed of some seventy so-called elements. The exact number does not concern us here; it will probably increase with the further improvement of our methods of observation. But the fact which claims our attention is that the apparently endless variety of all material phenomena, whether of organic or inorganic nature, are built up from so small a number of different substances —or rather that the number should be limited at all.

In what does the difference between these substances consist? The chemist provides some marks of distinction in his analyses—the color and form of the crystals precipitated by certain reactions, etc. In this way however only the most peculiarly characteristic properties are distinguished. The problem of the intrinsic nature of the different elements remains unsolved. Perhaps we may doubt if this question can be answered at all. In contrast to philosophy, natural science shows only that these distinctions exist, and that there are means by which they may be closely classified. Scientists are content to be limited to

* Translated from the German by Lydia Gillingham Robinson.

this classification; but even if they wished to go further, the line that divides the known from the unknown might perhaps shift its position but could never be removed. Nevertheless it may be profitable to attempt to formulate quantitatively at least the question in regard to the qualitative differences of matter; e. g. to reduce it to the purely geometrical question of form. Of course in analytical geometry every problem of form is easily expressed in terms of quantity, hence we would have the proposition comprehended with mathematical precision, and so at least simplified.

From time immemorial ideas in regard to the actual unity of matter have been generally current. Mechanics and astronomy deal with the concepts of weight and mass only, without having reference to any distinction between the various chemical elements. They presuppose as their foundation the idea of the unity of matter, wherein a common property of all matter is postulated.

The atoms of the chemist are the smallest divisions of matter that can either exist alone or can enter into combination with other atoms. This does not prevent them from being themselves composed of still smaller particles which, however, cannot be isolated. In fact one of the very latest beliefs is that even in this respect a step forward can be made; that a disruption of atoms may actually take place under the influence of electrical forces in the so-called Geisler, or Crookes, or Hittorf tubes; that the so-called cathode rays may prove to be nothing else than the paths of dispersed molecules. Even experiments with the mysterious radium seem to point toward the possibilty of so confining these ultimate atoms (*Uratome*) that one element may be changed into another, that out of radium, helium may be produced. The outcome of these experiments is still uncertain, but at least it is incontestable that these latest speculations in physical sci-

ence have succeeded in restoring and popularizing the conviction of the essential unity of all matter.

If the mathematician wishes to handle the proposed question with his own peculiar expedients he must first of all single out of the great variety of chemical properties, one that is accessible to quantitative representation, and which pertains only to the atom and not to groups of atoms such as crystal formations.

Each element has its particular spectrum consisting of a certain number of well-defined lines, by which the element may be positively recognized; i. e., in a gaseous and incandescent state (where the separate atoms circulate freely) each element, and therefore each single atom of the element, emits a light that when observed through the spectroscope does not furnish a continuous many-colored image, as for instance sunlight; but instead, a number of distinct lines whose position can be determined with extraordinary exactness, and each of which corresponds to a quite definite wave length of the particular light-vibration which exactly defines its own color. These wave lengths are numbers, hence quantitative sizes, by means of which all qualitative properties of the element concerned must be fully determined.

From these numbers the mathematician will proceed. He has the following problem before him: In space which is otherwise empty (occupied only by luminiferous ether) floats a material particle from which proceeds a series of vibrations each with clearly defined wave lengths, which are diffused in all directions and become visible as light of definite colors; what properties must this particle possess in order to produce light-vibrations having only these definite wave lengths or groups of wave lengths, and no others? The mathematical treatment is concerned only with the form of the particle and the distribution of matter within it, i. e., the density and elasticity of this matter.

But for such treatment the statement of the question must first be thus reversed: Given a particle of matter of a certain form, density and elasticity, that floats freely in luminiferous ether, what kind of light-vibrations originate from it? i. e., what are the wave lengths of the vibrations peculiar to it (the so-called *Eigenschwingungen*)?

So stated we have to do with a well-defined mathematical problem of the theory of partial differential equations; the matter composing the particle is set in motion (perhaps by contact with other particles); these vibrations continue in the surrounding ether throughout the surface of the particle without in any way interrupting the continuity in the motion of the wave through its surface; by this means is mathematically determined a definite or an infinitely great number of possible wave lengths, which produce an equal number of separate, distinct lines in the spectrum.

We must now vary the density, elasticity, form and size, in order to see whether through these can be reached the empirically established laws of distribution of the spectral lines of certain elements. Unfortunately the mathematical problem is so complicated that it is capable of detailed treatment only for a limited number of cases; and even then the equations are so extraordinarily intricate, that for the most part it must suffice to discuss the mathematical solutions only so far that the general type for the laws of the distribution of spectral lines may become apparent; and many elements too, on the other hand, are sufficiently characterized by this general type.

The simplest possible case is the following: The atom under consideration has the form of a sphere and is composed of matter of a given density and elasticity; what light-waves does this sphere generate? Here the calculation may be carried pretty far; the spectral lines are found to be distributed in the spectrum according to a simpler law than any to which an element has yet been

experimentally found to conform. Although it is enough in many problems of mathematical physics to represent atoms as spherical (especially in most problems of the kinetic theory of gas) yet atoms of an exactly spherical form are hardly ever met with.

Nevertheless the formulas which are valid for the sphere suggest a practically realizable result. When two spheres are composed of matter of equal density and elasticity but differ in size, we have the following law: If the lines of the spectrum for the light emitted by one of the spheres is known, those of the other may be found by multiplying the wave length of the known line by the ratio of the radii of the two spheres. Now if the spheres are of equal density, the ratio of the radii is equal to the cube root of the ratio of the weights of the spheres, or, in other words, the atomic weights of the two elements whose atoms are represented by the spheres. In this form then, the law becomes generally valid: The wave lengths of the spectral lines of two atoms of equal form, density and elasticity, similar to each other and differing only in size, are in the same proportion as the cube roots of their chemical atomic weights. According to this law the lines of one element may be computed from those of the other.

This law may easily be put to a practical test, and for this purpose it is necessary only to compare the spectral lines of different elements in the carefully prepared tables at our disposal. From this it follows that the spectra of the following groups of metals fulfill this law, either entirely or approximately:

1. Zinc, cadmium and mercury.
2. Magnesium, calcium, barium and strontium.
3. Silver, copper and gold.

These are groups of elements which are related in other respects because of their chemical properties. From their atoms we can tell that the elements of each group

are similar to each other; but on the other hand, similarity in chemical relations does not in the least imply similarity in the form of atoms. Proof of this is found in the case of alkalies (lithium, sodium, potassium, caesium, rubidium). The spectral lines of these related elements do not in the least conform to the law above mentioned. What then causes the difference between the three above mentioned groups? Either the atoms are composed of different substances or they are not similar in form.

Let us continue to consider the substance as identical, but vary the form. Instead of a sphere, let us choose first a prolate (that is, almost oval) spheroid, such as is generated geometrically when an ellipse rotates about its long

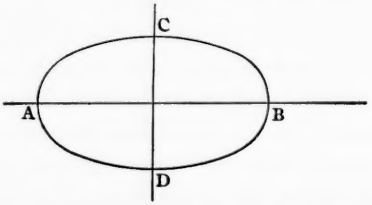

Fig. 1. This ellipse revolving about the axis A B generates a prolate spheroid, revolving about the axis C D an oblate spheroid.

axis. It follows from the mathematical theory that the spectral lines of such an ellipsoid depend on three numbers, that is they can be grouped according to three different principles. These numbers become the roots of certain transcendental equations, and are computed from the lengths of the axes of the ellipsoid, its density and elasticity—a calculation whose actual performance, to be sure, seems hardly practicable.

The first of these three numbers defines a group of related lines, a so-called series; a certain range of such series corresponds to the different possible values of the first number. The second number determines in each series a smaller group of lines, and the third points out in

each subordinate group one single individual line. The way in which the third number enters into the calculation, teaches us further that the reciprocal values of the wave lengths for the single lines of the above mentioned subdivisions, form constant differences among themselves, i. e., differences which depend solely upon the nature of the given ellipsoid. By this means we obtain a characteristic type of line distribution, a type which is familiar to us from the catalogues of spectral lines, and which is to be found only in the case of the above mentioned alkalies. For these Rydberg, Kayser and Runge have derived by observation certain conformities which essentially coincide with those for the spectrum of a prolate spheroid as above described, while the formulas obtained from experiments harmonize according to the type with those obtained from mathematical theory. The latter are derived from the representation of the solutions of linear differential equations by means of semi-convergent series. An actual calculation in all details is of course not possible as yet, because of the complexity of the formulas. The alkali atoms (Li, Na, K, Cs, Rb), we may inversely conclude, have the form of prolate spheroids, and for each single element the axis lengths of these ellipsoids are accurately defined; for different elements of this alkali group the ellipsoids in question are not similar to each other.*

An essentially different line distribution is produced by the so-called oblate spheroid. In this case, too, there are the same number of groups, series and subdivisions, but the law of constant differences becomes less universally valid; the roots of the transcendental equations become in

*The mathematical deduction of this and the following results have been published in two essays in the reports of the *k. bayer. Akademie der Wissenschaften*, Bd. 31-33 (1901-1903). The deduction of the more exact formulas by the aid of the semi-convergent series, as well as the consideration of the vibrations of rings and rolls (*Wülste*) may be found in a forthcoming sequel to those essays.

part imaginary; consequently some groups consist only of one single strong line, and others of a limited number. The more oblate the spheroid, the more prominent is the type. Experimentally the latter is known by the grouping of lines in the spectrum of the metals, gold, silver and copper. Hydrogen too, which in its reactions shows many analogies to metals, belongs to the same class, inasmuch as a thin round plate conceived as an extremely oblate spheroid, shows a spectrum of the same type as hydrogen.

In the third place we will consider the general ellipsoid of three axes, i. e., we will try to find the wave lengths of the light emitted by such an ellipsoid in an incandescent condition. The corresponding lines of the spectrum in this case likewise depend on three numbers, which are determined by transcendental equations and each of which can pass through a certain series of separate values. But these lines are not arranged in series and groups as in the two other instances, but instead are distributed over the whole spectrum. A series can be constructed only when the form of the ellipsoid approaches very near to that of an ellipsoid of revolution. This indeed can be discovered experimentally in the spectrum of the alkaline earths (barium, strontium, calcium and magnesium), i. e., those elements that in their chemical properties stand between alkalies and real metals. The same is true of zinc, cadmium and mercury. The former resemble more closely the prolate ellipsoid; the latter, the oblate.

If we proceed from a spheroid and think of it as varying so gradually that it becomes a general ellipsoid, then the lines of the spectrum will steadily change with the form of the atom, and it follows from the mathematical propositions that out of each individual line eight new ones will appear. Such a splitting of the lines, on the other hand, can be obtained experimentally by bringing the luminous atoms of the element under consideration between the

poles of a strong magnet; this is the well-known Zeeman phenomenon. Between the poles of a magnet the luminiferous ether is in a state of polarization, i. e., in a state of partial tension; but the mathematical treatment of the light-vibrations in polarized ether is the same for a spheroid as that of a general ellipsoid in unpolarized ether. Therefore we can make the inverse statement, viz., that the so-called Zeeman effect, or the splitting of the spectral lines by means of a magnet, is a consequence of the polarization of the ether and takes place as if the atom were correspondingly altered by compression.

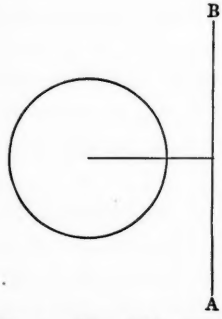

Fig. 2. The circle revolving about the axis A B generates the ordinary ring.

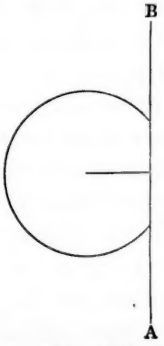

Fig. 3. This circle revolving about the axis A B generates the roll (*Wulst*).

Finally we will take a ring as a possible form of an atom for the basis of mathematical analysis, i. e., such a body as is generated when a circle revolves around an axis which does not pass through its center. We can distinguish two cases:

1. In which the axis does not intersect the rotating circle; the ring is open in the center and has the form of a round wire or rod bent into a circle, in short of a finger ring (Fig. 2);

2. In which the axis intersects the rotating circle; the ring is closed in the center; we speak then of a roll

(*Wulst*); the form is somewhat similar to that of an orange or an apple (Fig. 3).

The vibration problem may still be solved with regard to the ring form although the difficulties are increased; but with it closes the number of forms that can be successfully treated by mathematical methods.

The spectral lines of a luminous ring are dependent on four numbers, each of which must pass through a series of values, and indeed we can most simply get a clear idea of the type of spectrum by repeating several times consecutively the spectrum of a prolate spheroid, while at the same time of course, always shifting somewhat the relative position of the lines in the auxiliary spectrum.

If in the same way the spectrum of an oblate spheroid is repeated consecutively, such a system of spectral lines will be produced as would belong to a luminous roll of the kind above described.

Exactly in this manner Paschen and Runge describe the grouping of the spectral lines in the spectra of oxygen and helium on the one hand, and sulphur and selenium on the other; for according to their view the oxygen spectrum is produced when that of an alkaline metal is shifted repeatedly upon itself; and the sulphur spectrum, when certain groups of lines in the oxygen spectrum are replaced by single heavy lines. So we come to the conclusion that the oxygen atoms are probably in the form of a ring, and the sulphur atoms in the form of a roll.

There is a great fascination in following up the deductions which are connected with these notions and results. First of all it is noteworthy that the *chemical relation* of the elements seems to be dependent on the form of their atoms. Together with attraction and repulsion, the geometrical form of atoms must be considered in relation to the possibility and stability of a combination. So, for

instance, oxides of metals will result when the ellipsoidal metal atoms are deposited in the opening of the oxygen ring and close it up.

The water molecule consists of one atom of oxygen and two atoms of hydrogen; the latter we already know

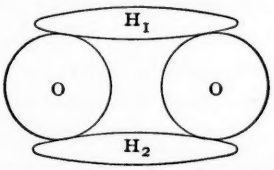

Fig. 4. Cross-section of a water molecule H^2O (one ring cut in two places O, O, and two oblate spheroids, H_1, H_2).

to be thin round plates. Hence when such thin plates take their places above and below the opening of the ring as it lies in a horizontal position, a water molecule is the result. Analogous to this is the construction of a sulphurated hydrogen molecule, for the roll of a sulphur atom exhibits a depression above and below in which a hydrogen atom can fit itself.

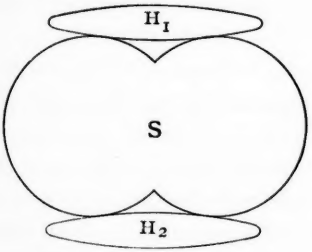

Fig. 5. Cross-section of a molecule of sulphurated hydrogen, H^2S (one roll S, and two oblate spheroids, H_1, H_2).

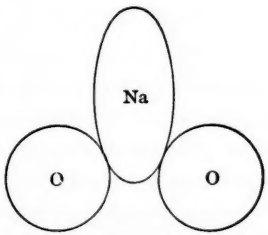

Fig. 6. Cross-section of a molecule of Na O (one ring cut in two places, O, O, and one prolate spheroid, Na).

Now the hydrogen atom may be replaced in every case by an ellipsoidal metal or alkali atom, which substitution would produce hydroxides and oxides, or on the other hand, metallic sulphides (for instance H^2S as shown in Fig. 5). But replacing one of the hydrogen atoms by a

sodium atom, there is no room left for a second sodium atom; consequently an oxygen ring can be combined with one sodium atom only (Fig. 6), and so one sodium (or alkali) atom is able to replace two hydrogen atoms. Under favorable conditions higher degrees of oxidation are formed while oxygen atoms again fix themselves upon the two metal atoms which then in turn offer free cavities for the union with still other metallic atoms.

According to this view, so-called atomicity is dependent on the form of the atom. Atomicity with reference to hydrogen (more generally with reference to ellipsoids) must be distinguished from atomicity with reference to oxygen (more generally with reference to rings). The former is equal to the number of the cavities of the atoms where an ellipsoidal atom may be joined, and the latter is equal to the number of prominences or convexities to which a ring can be affixed.

It would be of especial interest to study the chemistry of carbon combinations from this point of view. First would arise the question of the form of a carbon atom. Its spectrum throughout its entire extent is traversed almost uniformly by spectral lines, whence it is impossible to establish empirically any conformity to law and the mathematical treatment can therefore be of no service. Hence the question must be reversed, and the form of the carbon atom must be determined from the chemical nature of carbon combinations with other elements. This is a path which, as far as carbon is concerned, chemists have pursued under the leadership of Le Bel, van't Hoff and Werner, although with the other elements (with the exception of nitrogen) the question of atomic forms has not heretofore been raised. By this means organic chemistry has succeeded in essentially facilitating clearness of order and distinction in the endless variety of carbon combinations.

Different propositions have been made to so determine the form of the carbon atom that an explanation of the so-called isomeric combinations might be especially derived from it. Whether these propositions are consistent with the notion of atomicity here developed, whether they are not rather obliged to be modified or supplemented, cannot be discussed here. The question becomes intricate in the first place from the fact that the modern stereochemistry, the task of which is to demonstrate the construction of a molecule from its atoms in space, is satisfied by regarding whole groups of combined atoms as new units. When it shall come to pass that the atomic form of the carbon atoms is not used only as a symbol for the systematic arrangement of observations, but that the molecules of organic chemistry are really constructed in their spatial relations out of the form of all utilized atoms, then it will be necessary again to dissolve into its atoms every group of atoms in a molecule, which forms a definite smaller group. By this means the problem becomes unusually involved and we must be content to simply indicate it here.

By such a construction of the molecule a well-defined location within it is assigned to each atom, although it may retain enough freedom to be able to vibrate in this place around a particular point of equilibrium. We must, however, disregard the widely circulated notion that the individual atoms of a molecule describe definite paths around particular centers according to which each molecule would represent a miniature planetary system. This notion has been invented to justify the actual existence of an "internal energy" of the molecule from a mechanical point of view. This internal energy according to the theory here submitted, consists in the internal vibrations of the atoms which serve only in part as outward radiating beams of light, while others make their influence felt

as electrical or magnetic radiation, and at the same time repelling or attracting, so that they have an essential influence on all reciprocal action of atoms. Inasmuch as we treat of the so-called individual vibrations, i. e., of such as are transmitted without any discontinuity from the inside of the atom outwards to the luminiferous ether, the mathematical treatment teaches us that they can only spread simultaneously from the inside outward, and from the outside inward, and that whatever internal energy is lost will always be replaced from without, the luminiferous ether restoring this energy spontaneously. While this exchange of vibrations usually is not noticeable in most atoms under ordinary conditions and must first be aroused by warmth or by electrical energy from without, in the case of other elements (as radium, thorium, etc.) this exchange seems to proceed with extraordinary activity under customary conditions.

In another way the so-called electron theory undertakes to account for the internal energy of molecules. This takes for granted that a large number of electrons are bound up with every material atom; these electrons themselves are neither matter nor luminiferous ether, but a third something—electricity. This notion in addition to the laws of the transposition of ions has arisen from the practice of electrolysis and is based especially upon the remarkable phenomena of radiation displayed in the inside of Geisler tubes, particularly on the observation of the cathode rays. Often the electrons are looked upon directly as the ultimate atoms (*Uratome*) of which matter is essentially composed, the electrical forces dissipating atoms into these ultimate atoms whose paths then become the cathode rays. The lines of the spectrum are thus said to be generated by periodical vibrations of those electrons around certain points of equilibrium, while according to other views each electron has its own definite motion and

with it a definite spectrum whereby the number of lines in the spectrum would be equal to the number of electrons that are combined with the atoms of matter. At any rate a mathematical exposition of empirically determined laws in regard to the grouping of the spectral lines has never yet been attained on the basis of such hypotheses. In explanation of electrical phenomena this theory fulfills its purpose in many respects; as regards the cathode rays it does not contradict the views here submitted concerning the fixed character of atomic forms, for there is no reason why an atom of a particular form under suitable circumstances might not be separated into still smaller particles. What is really remarkable then is only the fact that the ultimate atoms can be accurately reconstructed; that after the electrical disturbance has ceased the original atom is formed again. The further pursuit of these remarks would lead too far into the theory of electricity for us to be able to engage in it now. It only remains to observe that the facts on behalf of the electron theory are also consistent with the hypothesis of atoms of definite formation. Indeed the admission that there are elastic vibrations within the atom is consistent with the notion that the atoms are in their turn composed of smaller particles which periodically change their relative positions by vibrations.

Our investigations have brought us to the point where the problem of the qualitative distinction between chemical elements is reduced to a mathematical problem of form and number in a series of instances. The individual element is characterized by the lengths of the three axes of the ellipsoid which represents the atom, or by the numbers which define the size and form of a ring or roll. Still unexplained, however, is the further question as to why only certain fixed groups of numbers come under consideration, only as many groups as there are different

elements with ellipsoidal or ring-shaped atoms. Why can we not choose these axis lengths and other determining quantities at will, and then assign to them the corresponding elements? Or in short, why is there only a limited number of elements? Why are the intervening links lacking?

Natural forces must exist which are still unknown to us and which operated decisively in the construction of atoms, perhaps are still operating uninterruptedly to-day, to preserve the atoms in their permanent forms or perhaps to prevent possible deflections and variations. This is quite analogous to that other question why only a definite limited number of living organisms exist or are capable of existing. We know that the struggle for existence and the survival of the fittest exercised a conspicuous influence on the formation and preservation of organisms; may not analogous considerations also govern inorganic nature? May not the existing elements have been forming themselves in preference to other eventual possibilities, just because they were best suited to their available external conditions?

These are questions which at present we can scarcely clothe in a mathematical garb. Much less are we able to furnish a solution until we succeed in so shifting the borderline of our understanding as to reduce the qualitative attributes of matter to quantitative differentiations. We may only trust that the future will dissolve and clear away the doubts that still remain.

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MANIFESTATIONS OF THE ETHER.

THE purpose of this article is to present to the non-technical reader a very brief review of modern ideas concerning the universal ether and some of its manifestations in the material world around us.

A few of the wise philosophers of olden times dimly imagined that there might be such a medium as the ether, yet little more than a century has elapsed since its existence was first seriously considered or admitted by a few advanced investigators, and even then only to accommodate and establish the undulatory theory of light. It is now recognized not only as the vehicle of light, heat and other radiant forces, but it is accepted by many scientists as the ultimate basis of the material universe.

This sweeping conception points to the absolutely immaterial origin of matter, and to its actual creation by superphysical means from this profoundly wonderful, unlimited, intangible and almost inconceivable entity,—the universal ether.

The phenomenon of light is one of its most familiar manifestations. We are accustomed to consider light as the antithesis of darkness, and the term is generally used to indicate the sensation produced in our organs of vision by ether waves of certain well-known frequencies and amplitudes.

All ether waves, however, which have the same physical characteristics as those which produce for us the vis-

ible spectrum may be classed with light waves, but it has become customary to generalize them under the common term of "rays," as for example, thermal or heat rays, ultra-violet or chemical rays, X-rays and so forth. There are other phenomena which have been also generalized under the same term although they are not produced by ether waves, but by the rapid flight of minute particles of electrified matter, or of electrons. In this category belong cathode and canal rays, also the alpha and beta rays which emanate from radium.

Etheric waves include a vast field of frequencies, beginning we know not how far down the scale, but first recognized in the electric waves of low frequency and great amplitude which were so brilliantly predicated by Maxwell in 1864, and afterwards discovered and demonstrated with exquisite skill by Hertz in 1888. Then sweeping upwards with ever increasing frequency and diminishing amplitude through a great unknown region, these ethereal undulations are next manifested in the heat spectrum which was so ingeniously explored and mapped out by Langley and other investigators.

Then passing from the heat rays, we see them dimly dawning in the first red beams of visible light, and now brightening and shining through all the rainbow hues of the visible spectrum until at length they fade into violet twilight and sink again into darkness. Yet not lost, but next manifested in the ultra-violet or chemical spectrum, and then through another vast and unexplored expanse to the region of the so-called X rays, and presumably onward from thence towards a vanishing point of infinite frequency and zero amplitude.

Considering the extent of the visible spectrum as one octave in this vast scale of etheric notation the invisible heat spectrum may be said to extend about five octaves below it. Still lower in the scale there are many octaves

of the vast unknown spectrum which exists between the longest heat waves and the shortest electric waves that are used in wireless telegraphy, etc. Above the single octave of visible light waves are found about two octaves of invisible ultra-violet or actinic waves. Then come seven or eight octaves of another great unexplored and unknown spectrum beyond which appear the X rays of varying qualities and probably the gamma rays of radium. These are believed to be manifestations of ethereal pulses having the highest frequency and the shortest amplitude thus far discovered, for of the region beyond nothing is yet known.

It is doubtful if we shall ever be able to comprehend the full compass of this vast and magnificent ethereal scale owing to the limitations of our perceptive powers, but enough of it has been revealed and understood to sharpen our desires for a broader knowledge and a clearer insight, and just as Maxwell was enabled to predict, and Hertz to produce and demonstrate the low frequency electric waves, and as Roentgen was led to discover and make visible the high frequency X rays, so will other investigators in this infinite field of research be inspired to unfold and bring to light many new manifestations that are now hidden from our view.

The tendencies of ignorance and barbarism are to scatter and divide, but the trend of advancing knowledge and civilization is to consolidate and unify. The early promptings of man's religious inclinations persuaded him to imagine that each object in nature was controlled by its own peculiar deity. He had "gods many and lords many" but with the development of higher mental qualifications, he became satisfied to believe in the existence and government of one God. So also many of the old philosophers believed in a multitude of various "virtues" or "emanations," each of which was supposed to be the origin of one particular natural phenomenon. Thus there was one virtue or ema-

nation that produced light, another heat, another magnetism, and so forth. In striking contrast to this heterogeneous multiplicity of scattered causes, let us consider the grand simplicity of the natural forces when demonstrated to us as only the varied but consistent manifestations of one universal medium. And yet who can feel surprise that these older ideas should have been conceived, for they were apparently based on principles of fact and common sense. How differently, for example, do light and heat appeal to our sense perceptions, and who would naturally suppose that each of these sensations is caused by the same order of ethereal undulations? So also with the electric waves of Maxwell and Hertz, and Roentgen's X rays, to all of which our senses are impassive and which are only revealed to us by their secondary effects. How could any superficial observation of these phenomena inform us that they are all due to a series of ethereal waves which differ only in magnitude and frequency from other waves that convey to us the sensations of light or of heat?

Having thus very briefly considered some of the natural forces as manifestations of the ether, let us now turn our attention to tangible matter.

Within comparatively recent times the indestructibility of matter was hardly questioned, and the absolute permanence and indivisibility of the material atom was generally acknowledged. New elements were being discovered by the chemist and we were growing somewhat confused by the ever increasing number of presumably unalterable and eternally permanent elementary substances. For some obscure reason the scientific mind often rebelled, at least inwardly if not openly, against this ever growing complicity in nature and yearned for basic simplicity and unification. How pleasing and satisfying therefore is the prospect of solving this perplexing question by the revelation of a material monism that is based upon a rational

interpretation of indisputable facts, and which appeals with equal force to both reason and idealism.

The so-called elementary substances were long ago separated into their constituent molecules and the molecules were divided into atoms, but here we came to a standstill until the electronic hypothesis suggested the subdivision of the atom into its ultimate particles. These transcendently minute entities which Dr. Johnstone Stoney very aptly termed "electrons" have been most carefully and diligently studied, but it is possible that their structural mechanism may never be clearly determined and understood. It is however probable that they are simply differentiated ether whirls or vortices that are endowed with intense activities of unknown, perhaps unknowable, origin. The material atom, far removed from individual inspection by reason of its infinitesimal size, is yet of colossal proportions when compared with the inconceivably minute ethereal monads which are believed to perform their orderly planetary evolutions within its boundaries, and the suggestion is offered that it is their relative numbers and their respective groupings and velocities, together with the scope and general characteristics of their orbits, that individualizes their containing atom, which in turn determines the physical qualities of the so-called elementary substance of which it may form a part.

The interatomic spaces between these electrons may be comparable to planetary spaces, and it is at least conceivable that the mechanics of the visible universe may be typical of atomic structure, making the infinitely great emblematical of the infinitely small, and thus, quoting the words of Professor Serviss, "Starting from the smallest things of which we can have any knowledge and rising to the greatest things which the universe presents, we may find everywhere a single plan running through them all,

the stupendous star cluster showing us an image of the infinitesimal atom."

According to this hypothesis, therefore, the entire material cosmos may be considered as simply a manifestation of the ether. Ether points endowed superphysically with various activities appearing as electrons, systems of electrons of more or less complexity, forming atoms, atoms combining into molecules, and molecules building up matter in all its various physical forms and countless combinations. So also, on the other hand, motions and strains communicated to the surrounding ether by atomic or interatomic activities are manifested in the various natural forces, which in their turn again affect other material atoms or their constituent parts. The unending cycles of cause and effect are thus continued, and we have found that although when at rest and under no strain, the ether appears to us as nothing, yet when endowed with motion and under strain it becomes everything.

All substantial matter and all intangible forces, all things in fact of a physical character that our senses can perceive or our minds can contemplate are resolvable into manifestations of this wonderful medium. Thus far has science seemingly traced back matter and energy to their sources: ether and motion, but here the limit of human understanding is reached. The First Great Cause of physical phenomena is still beyond us and we are again confronted with the time-worn question, "Canst thou by searching find out God?"

These far-reaching conceptions which are alike attractive in their beautiful simplicity and their mysterious depth, point to the possible explanation of some hitherto troublesome problems. For example, they entirely dispose of that logical absurdity the solid indivisible and indestructible atom, and at the same time illustrate in a rational manner how the most minute of material entities

may have a conceivable origin. They also throw some light on the mysterious force of gravitation, resolving it into an ether strain dependent on the interrelation between the spinning systems of ethereal monads and their containing medium.

The activities of these cycling electrons being in general ungoverned by known physical laws would account for the apparently unchangeable nature of so-called elementary bodies. Yet we know that at least some of these bodies are not unchangeable, for the discovery of radio-active matter has revealed the extraordinary fact that certain elements appear to undergo a continuous process of spontaneous disintegration and subsequent reconstruction into other elements of more simple physical constitution, as shown for example in the gradual evolution of helium from radium, which will be referred to again later on.

To all commonplace intents and purposes however that which is usually termed elementary matter may be considered stable, because atomic changes are so inconceivably gradual and minute that they cannot be measured, excepting by indirect and very difficult methods, and even then only in connection with a few substances which are remarkable for their high atomic weight and strong radio-active properties.

To illustrate the exceedingly minute character of these atomic changes and to show how utterly impossible it would be to detect them by any ordinary means, it is stated on good authority, that in a small scrap of radio-active material, hundreds upon hundreds of atoms may be continuously disintegrated every second and yet this process may continue in operation for hundreds and perhaps thousands of years before any appreciable change in weight could be detected. Moreover, we are assured that atomic disintegration is constantly proceeding in radium and some other substances, and it is legitimate to infer that

this action is not confined to obviously radio-active bodies, but that gradual change, at a much slower rate, is the common fate of all matter, which may therefore be broadly considered as only a transient manifestation of the ether, although it may appear to us as eternally permanent.

It is well known that the ordinary flame spectra of many elements, both solid and gaseous, may be modified or entirely changed under the influence of the electric arc or condensed electric discharge, and this fact, strengthened by additional evidence found in the spectra of the sun and of certain stars, has pointed to the resolvability of these elements into more simple constituents under certain conditions. Metals and other heavy elementary bodies however, if vaporized by intense heat, will solidify into their original form again on cooling, and elementary gases when condensed into liquids or solids by the action of extreme cold and enormous pressure, will expand again into their original state, with their qualities unchanged, when normal conditions are restored, thus indicating in each case, that the mechanism of the individual atom has not suffered any permanent disarrangement.

Neither heat nor cold nor any other physical influence, in degrees that are attainable by us, can apparently penetrate or affect these foundation stones of matter, for the forces at our command are incomparably weaker than those which control and maintain the integrity of the elemental atom.

The life history of a chemical atom may cover a period of existence which exceeds the millions and millions of centuries occupied by the birth and decay of a planetary system by as much as the latter exceeds one short moment of our existence, and it is difficult for our finite minds to form any adequate conception of these tremendous eons of duration.

Material bodies however do not consist only of aggregations of atoms, but contain also free electrons, although to what extent has never been clearly determined due to their possessing few, if any, of the physical attributes common to gross matter. There is reason to believe, however, that the apparent transit of electricity through conductors can only be effected by the migration of free electrons, and for this reason we may infer that in metals and other substances which conduct electricity freely, they must exist in vast quantities and also be in a very mobile condition. The hypothesis has indeed been advanced that the conductivity of metals may be directly proportional to the extent of the mean free path of the electrons which swarm within the interstices between their atoms. On the other hand it may be legitimate to suppose that bodies which do not normally conduct electricity contain few electrons, or rather that the electrons within their substance may be bound or fixed in a way which checks their mobility of action. The last hypothesis appears to be the more feasible because some substances which are non-conductors at ordinary temperatures acquire conducting properties when strongly heated.

It is not within the scope of this review to discuss at any length the intimate relationship which appears to exist between electrons and that *something* which is manifested to us as electricity, but it may not be out of place, to touch very briefly and superficially on this subject.

Electrons can be expelled from matter in many ways, and in every case they appear to carry with them a negative charge of electricity, leaving the substance from which they have been subtracted positively charged, but free electrons carrying positive charges have never been isolated. When matter is negatively charged it is therefore presumably due to the assimilation of free electrons. Attention may here be drawn to the anomalous fact that a so-

called positive charge of electricity is due to a deficiency of electrons, and a so-called negative charge is the result of their surplus accumulation. This reversed nomenclature, although a little unfortunate, does not affect final results in practical work, so the terms "positive" and "negative" are still commonly used in accordance with the original application given to them by Benjamin Franklin, and it is interesting to note that although he accidentally reversed his plus and minus signs, yet his so-called "single fluid" theory of electricity borders very closely upon our most recent conceptions.

Seeing that these modern views of electronic mechanism are largely due to the discovery of radio-activity, or at least that they have been greatly strengthened and reinforced thereby, a few remarks on the most important features which have led up to these profoundly interesting and important results may be acceptable.

Radio-activity was first observed as a new property of matter by Becquerel about nine years ago at the time when the then recent discovery of X rays by Roentgen had naturally stimulated scientific research. It had been generally known for some time previous to this that certain minerals possessed the property of absorbing or storing up light rays and afterwards emitting them as a phosphorescent glow visible in the dark. The fact that X rays are able to affect a photographic plate through substances that are opaque to visible light suggested the possibility of sunlight being absorbed and then emitted in some form of radiation similar to X rays. Becquerel had been experimenting along these lines and he had wrapped some sensitive photographic plates in folds of black paper and placed them in the sunshine with lumps of pitch-blende over them. The story goes that a sudden rain storm caused him to hastily bring them under cover and they were put away in a dark closet and forgotten for a day or

two. Becquerel then brought them out to expose them to sunlight once more, but thought he would first try to develop one of the plates to ascertain if the original short exposure had produced any effect. To his surprise he discovered that the lumps of pitch-blende had photographed themselves on the plate with a strength that appeared to be out of all proportion to the brief time of their exposure to sunlight. Becquerel's acute mind immediately suggested to him that some action must have taken place in the dark closet, and on repeating the experiment without any exposure to light, he found that the pitch-blende produced a photograph of itself on the sensitive plate through several folds of black paper, when kept in perfect darkness. This was certainly a wonderful clue, and it was the starting-point of a series of experiments with which the world is familiar, and which have immortalized the names of the Curies and other investigators whose labors have revealed to us the wonders of radio-activity as exhibited in radium, thorium, uranium and some other kindred substances.

Selecting radium as a representative of this class, its most interesting and important characteristics may be summarized as follows:

1. It has the property of affecting a photographic plate through certain materials which are opaque to visible light.

2. Its emanations will discharge an electrified body by ionizing the surrounding air and thus making it a conductor. This feature is so marked that it furnishes the most delicate test known for the detection of radio-activity.

3. It excites fluorescence in certain minerals, but this phenomenon is partly of a different character to the ordinary fluorescence or phosphorescence that is excited by their exposure to light rays, as will be noted later on.

4. It generates continuously an appreciable amount of heat, so that under proper conditions it will maintain a

temperature within itself two or three degrees higher than that of surrounding objects.

5. It has the property of imparting a temporary radio-activity to all substances in its neighborhood by the deposit thereon of an infinitesimally thin layer of radio-active particles derived from its emanation.

6. It has a very marked effect on living tissue, and tumors, ulcers, cancers and other malignant growths have been pacified and even cured by its application.

Dangerous burns may also be inflicted by using it indiscretely, and it is a curious fact that these effects, like burns from X rays, may not appear immediately, but may develop many days or even weeks after the application has been made.

We may now proceed to briefly consider the origin or cause of some of these extraordinary properties, drawing our information from the classical researches of Ramsay, Rutherford, Soddy, and a few other scientists who have given special and untiring attention to these matters.

Referring to previous remarks on the mechanism of the atom, it is conceivable that certain conditions may occur to disturb the equilibrium of the velocities and orbits of an electronic system constituting an individual atom, in which case it may become unstable and explode or fly to pieces, either partially or entirely.

The atomic weight of radium is believed to be about 225, and when one of these heavy atoms is disintegrated, a train of results will follow which point to a partial explanation of its remarkable properties.

1. Material atoms that have been commonly termed "alpha" rays or particles are projected from the disintegrated parent atom with varying velocities up to a maximum of about 20,000 miles per second. These particles are not radio-active; they have about twice the atomic weight of hydrogen atoms and they are believed to be in

fact atoms of helium. Although they are projected with a speed that far exceeds that of any other known form of moving matter, their velocity is slow when compared with that of free electrons, and they possess but slight penetrating power, being unable to pass through even a thin piece of paper. When they impinge on certain materials, as for example, fluorescent zinc sulphide, they produce little flashes of light. The scintillations thus produced are so numerous and minute that in the aggregate they present a faint phosphorescent glow to the unaided eye. On examination, however, with a magnifying glass of moderate power, say about twenty diameters, individual scintillations become visible and appear as a multitude of constantly changing and twinkling points of light in a dark or slightly luminous field. Sir William Crookes was the first to discover this beautiful phenomenon and he applied it to interesting service in his well-known spinthariscope.

2. The residuum of the parent atom after its primary disintegration and the projection of "alpha" particles, assumes the form of an intensely radio-active emanation or heavy vapor, which continues to spontaneously decompose, ejecting more "alpha" particles, and breaking up rapidly in successive stages into other more or less radio-active products, each one differing in some degree from its predecessor.

3. Free electrons are also expelled from these products, which are endowed with enormous velocities varying from 60,000 to 175,000 miles per second, thus, in their most rapid flight closely approximating the speed of light rays. These electrons always carry a negative charge of electricity and they are commonly termed "beta" rays. They were foreshadowed in Faraday's "radiant matter" and are believed to be identical with the cathode rays formed in a Crookes' tube. Each electron appears to possess a uniform mass of about $^1/_{770}$ of a hydrogen atom and collectively

they have considerable penetrating power, their intensity being only reduced about 50 per cent. by passage through two inches of air at atmospheric pressure or $^1/_{50}$ of an inch of aluminum. The tremendous velocity of these "beta" rays or free electrons, and likewise that of the comparatively massive "alpha" particles points to the almost inconceivably high orbital speed of the members of the electronic systems composing the parent atoms from which they are projected when these atoms are disorganized and broken up by interior disturbances.

4. Incidental with the expulsion of "alpha" particles and "beta" rays, ethereal pulses are propagated with the velocity of light rays and possessing strong penetrating power. These ethereal pulses are termed "gamma" rays and they are probably produced by the impact of the "beta" rays against the solid material of the radium compound in the same manner as X rays are evolved by the impact of cathode rays against the platinum target of a Crookes' tube. Their resemblance to X rays is further demonstrated by their power to produce fluorescence in platino-barium cyanide, a screen of which material gave to Roentgen his first accidental clue to the discovery of X rays.

5. After undergoing sundry transformations as already noted, with the evolution of "alpha" particles, free electrons, and "gamma" rays, the material is probably reduced eventually to some stable substance or substances, and the universal presence of lead in ores containing radium has suggested that this metal may be at least one of the final products.

Radio-active effects may therefore be classified under several different headings such as, for example, thermal, luminous, chemical, electrical, etc., thus corresponding with the manifestations of ethereal waves before referred to, although not confined within these limits.

To sum the matter up, these effects are believed to be the result of a spontaneous disintegration or breaking up of heavy and complex atomic structures into lighter and more simple atoms and free electrons, accompanied with the evolution of energy which, considering the small mass of matter involved, has been estimated by Rutherford to be at least a million times greater than that developed by the most violent chemical reaction; comparatively stable matter being eventually formed as a last residue.

It is not consistent with sound and progressive thought to believe that present views are infallibly correct. They may indeed be considered extremely crude in some respects and will doubtless be subject to many changes in the future. We trust, however, that our progress towards a realization of ultimate truth will be hastened, and that we may be led onwards and upwards to clearer conceptions and more intimate knowledge of the mechanism of matter and the manifestations of the universal ether.

I may fittingly conclude by quoting the words of Lord Kelvin, who for more than half a centruy has devoted much time and thought to the solution of these difficult problems. He says: "Overpoweringly strong proofs of intelligent and benevolent design lie around us, and if ever perplexities—whether metaphysical or scientific—turn us away from them for a time, they come back upon us with irresistible power, showing to us the influence of a free will through nature and teaching us that all things depend on one Everlasting Creator and Ruler."

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HEREDITY, AND THE ORIGIN OF SPECIES.*

THE surface of the earth is inhabited to-day by hundreds of thousands of forms of life, which upon analysis are found to be separable into groups or species, with well-marked and characteristic attributes, which may be transmitted from generation to generation. The strata beneath the surface are found to contain the remains of thousands of other forms, now extinct, but which show certain general relationships to the existing organisms. If we piece together the actual records, and take the backward bearing of our information, we arrive at a period sometime within the last six hundred thousand centuries, when living matter was more indeterminate in the forms which it assumed, and was, perhaps, quite unlike any protoplasm we meet at the present time.

From this primitive substance series of organisms have been produced, which, in the successive stages of the earth's history show an increasing complexity as the present epoch is approached, and which embrace more numerous forms with the advance of time.

In this upward movement, this evolution from the simple to the complex, in the production of many from few, it is not to be taken for granted that protoplasm has been a perfect automaton, and that it has nothing but successes scored to its credit in the ever-changing conditions it has

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met since its beginning. On the contrary the succession of its forms is a most devious one, and by no means easily traced. Phylogenetic systems are made up chiefly of allowable suppositions, and it is altogether probable that we do not know more than a fraction of the course of the zigzag, halting, and at times, receding steps followed by living matter in its development into existing types.

Thus the mosses are seen to have reached the ultimate development allowable by their morphological character, and hence are incapable of any movement which will produce higher or better organized types. Accordingly we may assume that their existence as a constituent of the flora seems to be doomed to a diminishing importance, if not final extinction. The fern-like plants are also examples of a great phylum of the vegetable kingdom which, in the Carboniferous period, being most suitable for the conditions present, constituted the prevailing type of vegetation, making up perhaps as much as three-fourths of the plant-covering of the land. Their organization is one which promises nothing of advantage vegetatively and reproductively, and we are now at a time when we see them slowly but inexorably, being supplanted by seed-producing forms.

With the advance and retreat of the ice-sheets, the upheaval and submersion of land areas, and other great alterations in the earth's surface the hosts of plants have marched and countermarched across the face of the world, now losing thousands of species, the remains of which we exhume from ancient battlefields; now entering upon new possessions and increasing the numbers of species and forms by originating new types, which survived or perished according to their fitness or unfitness. In the interpretation of the traces of these major movements of living forms great care must be exercised in distinguishing the essentially different factors affecting the distribution of

plants and animals. It is as if two games of checkers were being played on an enlarged board at the same time, the moves of each player being governed by those of his opponent and interfered with by those of his neighbor, and the two players on one side of the board by no means advance, guard and retreat in parallel fashion, while their losses and gains may be wholly disproportionate. With regard to the application of this warning note it is to be said that the "naturalists," as some zoologists term themselves, having made the greatest number of essays to offer a universal interpretation of the problems of distribution are to be credited with the greatest number of defenceless assumptions. In all developmental and evolutionary researches too much emphasis can not be laid upon the basal fact that the physiological and morphological nature of the two great classes of living things are so widely divergent, that the derivation of universal biological principles from their apparently concurrent behavior, must be made with the greatest caution.

Let us pass at once to the basic problems of life, those of the origin of characters and their inheritance, by which new species arise. Very naturally the first statement advanced was a purely theological one, that species were created directly, and that we need not expect to find an explanation in natural causes.

Of the hypotheses that may be taken more seriously, one that has even lately been regarded with much favor, predicates that organisms undergo transformations, or alterations by adaptive changes in their organs and entailed functions, as a direct reaction to environmental factors, and the altered features becoming fixed and inheritable, the organism receives a lasting imprint from its habitat. Migration to new areas, or changed climatic conditions might give opportunity for new stimuli and different reactions which might, or might not, affect those pre-

viously made. We need not, at this time go into the intricacies of the arguments that cluster about this main thesis, or weigh the evidence that is drawn from the presence of useless, vestigial, and useful characters in various organisms.

Popular belief in the influence of environment and the inheritance of acquired characters finds its commonest expression in "that plants have been changed by cultivation." Domesticated races are spoken of as "garden forms" by botanists and horticulturists, with the implication that they are specialized types resulting from the effects of tillage. Now so far as actual cultivation is concerned, this assumption is without foundation, since at the present time no evidence exists to show that the farm, garden or nursery has ever produced alterations which were strictly and continuously inheritable, or were present, except under environic conditions similar to those by which the alterations were produced, although vague statements and erroneous generalizations to the contrary are current. It is true of course that structural and physiological changes may be induced in a strain of plants in any generation, which may persist in a share to the second, or even in some degree to a third, but no longer. Some very important operations of the market gardener and the farmer are dependent upon this fact.

The possibility that permanent changes might be induced is by no means to be denied, and it is a fair subject for investigation. Actual evidence is to be obtained only by observations of breadth under guarded conditions. Until this is at hand all affirmative conclusions can be but inferential and suggestive.

Vicinism, the somatic multiplication of bud-sports and extreme variants from a fluctuating series, and the confusion of closely related elementary species form the basis for the greater number of mistaken assertions as to

the effects of cultivation. It is obviously necessary to examine all facts bearing upon the lineage of supposedly new forms, with the greatest care before their aspect, or behavior may be taken as evidence upon phylogenetic problems.

The theory of natural selection of an intra-specific application, as one of which new forms might arise, is so well known that we need not particularize in defining its ramifications. Briefly, it is assumed that as the whole mass of individuals comprised in a species is in a constant state of variation the individuals which show features even the most slightly better adapted to the environment, survive while the less fit perish. Thus by infinitely small changes during each generation a species moves away from the ancestral type in one or more directions until in the course of thousands of years the differences become so great as to be appreciable, and of a specific character, to use an arbitrary phrase. Three unsurmountable objections to the acceptance of this method as universal present themselves. First, the fluctuations exhibited by the individuals comprised in a species do not in any known instance transgress definite measurable limits, and do not depart from an ascertainable norm or average. Secondly, the gradual transition of individuals from one type to another, that is intergrading forms, demanded by this theory, are not found among plants, and thirdly, although we have preserved specimens and records of several species which cover their history for many thousands of years, yet such gradual transformations are not observable. Lastly it is to be said that it is extremely doubtful if the earth is old enough to have permitted the development of the great number of organisms which inhabit it, by this method.

A secondary idea that has been formulated in connection with this subject is that of orthogenesis, by which the organism evolves rudimentary structures purely as a result of internal forces, and initially without reference to util-

ity, or to environment. These rudimentary organs may, in the course of generations wax in size, undergo increasing differentiation of structure, perhaps finally becoming sub-functional or even fully functional, although nascent organs are not always supposed to attain this useful end. In support of this theory it has been pointed out that plants and animals show many structures, which so far as our understanding of them goes, are wholly without part in the life of the organism. The present development of plant morphology, however, is one which is carrying us farther and farther away from the conception of such pre-functional formation of organs, as the whole tendency of modern investigation is to place the morphogenic processes upon a physiological basis. On the other hand the argument that the variations of the organisms are "determinate," and are governed by the morphological possibilities is one which surely holds for any method of phylogenetic procedure. Thus it needs but the briefest common-sense consideration to show that any given type of leaf or flower, could not possibly vary toward all other types, but only in the direction of certain forms not too widely different.

A more explicit statement may prevent a misconception. It has been held by some writers that variations and mutations, theoretically follow those already made, as a projection of them, or a continuation, which carries the organs concerned, successively and ever nearer some ideal form or type. This is determinate variation in its strictest sense. On the other hand, it is argued that from any given stage in its development an organ may vary or mutate in any direction limited of course by its morphological possibilities, and such alteration may lead the structures concerned in any given course from that previously pursued. It need only be said that to the experimenter the latter view seems to be the more fully justifiable by the facts observed in mutations.

Of the many other phases of the existence, and distribution of organisms, that have been considered in a controversial way to prove this, or that, hypothesis it must be ceded that in this discussion of the effect of isolation upon the origin and preservation of forms, the evidence adduced, so far as plants are concerned does not especially favor any of the suppositions discussed previously. In like manner the bogie of the effects of close- and cross-breeding is used in the most reckless manner to support the most diverse generalizations. Until investigated with the exactness demanded by modern technique, and by the most approved methods with material of undoubted pedigree, we may well consider close- and cross-fertilization as operations the value of which is yet to be estimated. Nearly all of the available data have been obtained with domesticated races, artificially selected from what, in the natural conditions, may be the most extreme phase of a character, and far from the norm of the species in many particulars. The retrogression toward this norm may be a deterioration of the economic value of the strain, yet from the point of view of the species itself it may be a direct movement toward the average condition, in which we may not secure so much speed, beef, wool, milk, grain, or fibre, as in strains bred and selected for these features.

It is with great relief that we turn from these historical and hypothetical considerations to the authenticated facts, the interpretation of which affords some positive generalizations as to phylogenetic procedure. In other words we take up the observations by which species *have* arisen, in place of the suppositions as to how they *may have* arisen.

First, it has been known for over half a century that fixed forms, constant in inheritance and self-maintenant, therefore constituting species have resulted from hybridization. The fertilization of the egg-cell of one species by the pollen of another, often results in an interlocked and

stable combination of the characters of the parental forms in such manner as to give rise to a new type unlike either of the parents, variously intermediate and constant to the new type in succeeding generations. More than a thousand such fixed hybrids, or hybrid species are known, some of which have been formed anew experimentally and are thus beyond doubt. It is to be seen therefore that hybridization has played, and is playing, no small part in the composition of the flora of the earth, and that it must be considered as an active, and not unimportant factor in the evolution of plants.

It must be admitted at once that such fixed hybrids are exceptions to accepted principles of dominance, and of the purity of the germ-cells, as the fixed hybrids depart entirely from the Mendelian procedure. As a further complication we have in some instances a hybrid progeny comprising several fixed types, each constant to itself. The only possible explanation of such phenomena in the simpler case calls for a fusion of the germ-cells and a double longitudinal division of chromosomes.

Such splittings are qualitatively unsymmetrical, since the fixed hybrids are variously intermediate between the parents, sometimes being goneoclinic to the one furnishing the pollen, and in other cases to the one bearing the egg-cell. When it comes to the composition of a hybrid progeny which comprises several fixed types in the first generation, breeding true in succeeding generations, we can only look to the researches of Wilson and others upon the quantitative and qualitative inequality of the chromosomes to furnish us the proper clue. It is only fair to say that no investigation of such cases has yet been made. Now as to a third category in which the hybrid is polytypic in the first generation, and some or all of the types may split in the second generation into forms, which may or not be identical with the grandparents, we evidently have a com-

plex case in which the partial fusion of unequal chromosomes, with divisions of the chromosomes may operate in a manner to secure anything but purity of the germ-cells. The results of such complicated integrations and splittings are incidental to the scope of this lecture however, as we are concerned at present only in those cases in which fixed forms result from hybridization, chiefly in the first generation. So far have we advanced, however, in the study of the behavior of hybrids, that with the information obtained from them, and with the recent discoveries as to the differences of chromosomes, we may well believe that we are now at the dawn of a new day in the study of heredity and all of its associated problems, in which the greater results are to come from the proper and closer correlation of the researches of workers in all branches of natural science.

The second method by which new characters and new species have been seen to arise in the succession of generations in plants is that of discontinuous variation, or mutation. In following out the germination of hundreds of seeds in pedigreed strains of plants, a few individuals may be found in each generation, which are notably different from the type in anatomical and physiological features. These divergencies are variously heritable, constituting breaks in descent, by which, in some cases, new species originate. Such seed-sports or seed-mutants have been seen hundreds, perhaps thousands of times during the last two centuries, but it is only within the last twenty years, that their phylogenetic importance began to be appreciated. It will be of interest to this assembly to know that American botanists and horticulturists have made early observations of direct interest in this connection. Thus Dr. Arthur Hollick, while a student at Columbia University in 1879, said in writing of white varieties of colored plants: "First then, we have to consider those sports of nature where

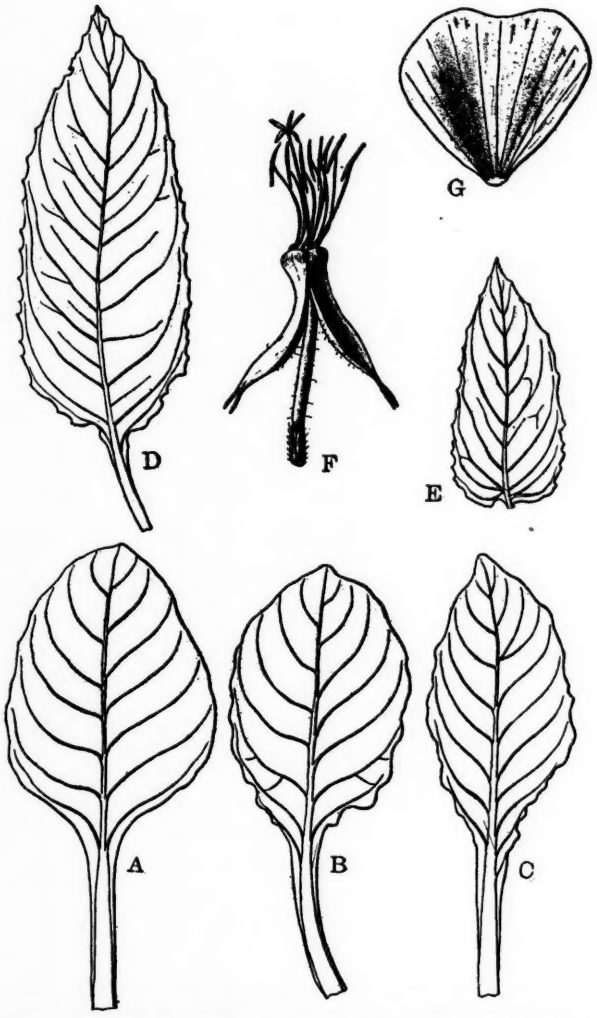
there has been a sudden change, without any intermediate steps, from a plant with colored flowers to a pure white variety: such may be termed "negative" varieties, since their peculiarity is due rather to an absence of color, than to the presence of white. Not only does the flower show the characteristic absence of color, but the leaves, stem, and, in fact, the entire plant, are invariably of a lighter green; and if any red be normal to the stem (which is often the case), this will also be of a lighter shade. It has often been urged that these albinos are mere 'sports' of nature with nothing constant about them,. . . ., and that there is nothing inherent in the constitution of the plant. Fortunately I have been able to test this,. . and found them, . . not only constant in their peculiarities, but also that these are bred in the plant and capable of inheritance." Three years later, about the time that De Vries began casting about to find material suitable for the demonstration of his theory of unit-characters and their saltatory action, Mr. Thomas Meehan, a horticulturist of Philadelphia, wrote as follows, in a discussion of some anomalous form of the oak: "The conclusion that I have been forced to is that the odd forms we often find in nature are not necessarily hybrids, but are as likely, if not more likely to be the outgrowth of some internal law of form with which we are as yet unacquainted. That they do not often perpetuate themselves is (*not*, plainly implied) remarkable when we remember that of thousands of seeds produced on any one tree, but a very small percentage ever gets a chance to form, and of those which do sprout, again but a small percentage survives to become bearing trees. As the number of trees reproducing the general features of the original may be as a hundred to one of the more strikingly aberrant forms, we may see that though individual instances may be common, we are never likely to meet many trees of one stamp. Once in a while an indi-

vidual tree may find itself in a situation favorable to the preservation of a number of seedlings, which might endure until again reproductive; in such cases a marked variety may originate and make its way over the earth.

"I have often thought it probable that in time a few individuals of these suddenly introduced forms might again leap into new features, and then if they should be able to sustain themselves, we should have new species quite independently of any principle of natural selection; that principle, as I understand it, being governed chiefly by 'environment.'"

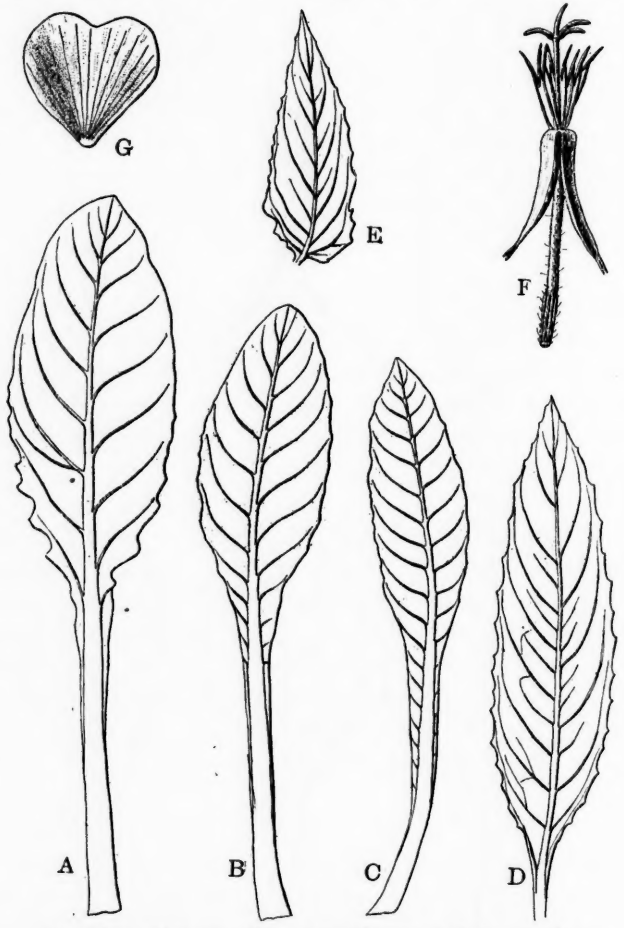
These utterances may be taken as prophetic in part, and in part as a natural expression of the inadequacy of explanations of the origin of species current at the time. Shortly afterward, Professor De Vries, impressed with the necessity for obtaining positive evidence upon the subject, began an examination of the plants in the vicinity of Amsterdam, Holland. Over a hundred species were brought into cultivation, and tested by guarded pedigree-cultures, and every precaution was taken to exclude interference of agencies which might introduce errors. It is impossible to describe the enormous amount of work entailed in such investigations, but the truly splendid results well justify the tedious care by which lines of descent were carried through successive generations for two decades without allowing a trace of doubt as to the purity of the lineage involved.

During the last four years a partial duplication and an extension of these experiments has been carried on in New York. It seems unnecessary at this late date to go into the detail of the mutations of Lamarck's evening-primrose, especially since the phenomena exhibited by them are also to be seen in many other species. By way of illustration it need only be said that this plant, as originally observed by De Vries, was found to give from two



Oenothera Lamarckiana, the parental form which is giving off many mutants. A, B, and C, leaves from rosette. D, leaf from middle of flowering stem. E, bract. F, flower with petals removed. G, petal.

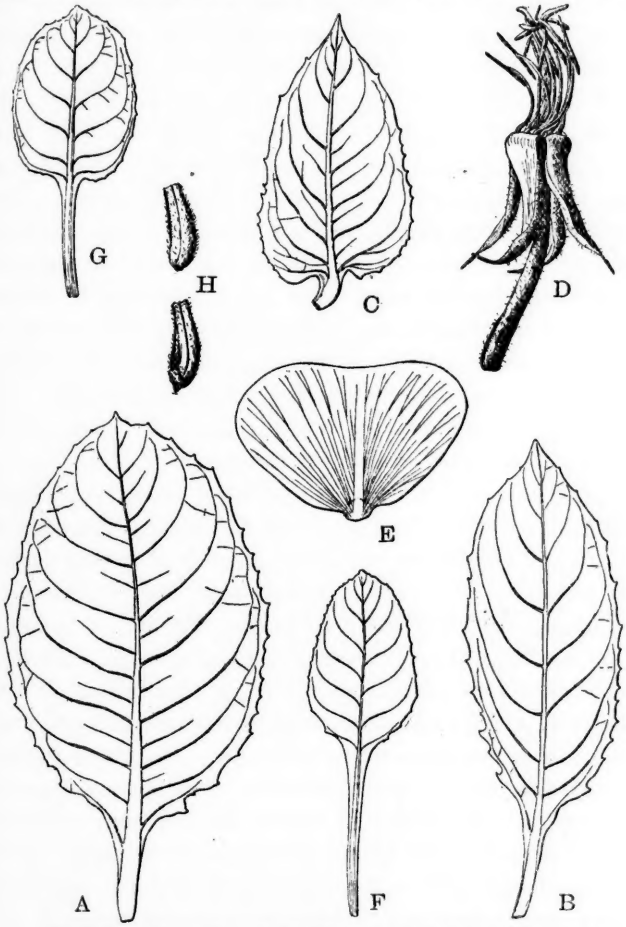
to five plantlets in every hundred which were widely divergent from the type, and could, upon maturity, be ar-



Oenothera rubrinervis, One of the Mutants of *O. Lamarckiana.* A, B, and C, leaves from rosette. D, leaf from middle of flowering stem. F, flower with petals removed. G, petal.

ranged in several groups, and in fact constituted several possible species. One, *scintillans,* was eversporting, and

continuously and consistently gave rise to a variety of forms in its progeny, which included the parent and its

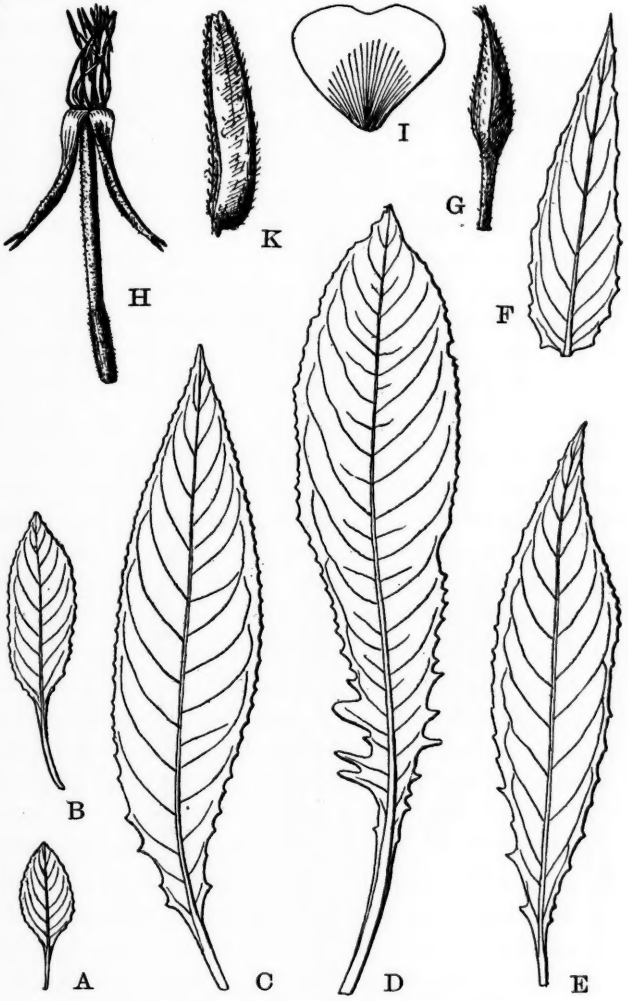


Oenothera gigas, One of the Mutants of *O. Lamarckiana*. A and B, leaves from middle of main stem. C, bract. D, flower with petals removed. E, petal. F and G, leaves from the rosette. H, capsules.

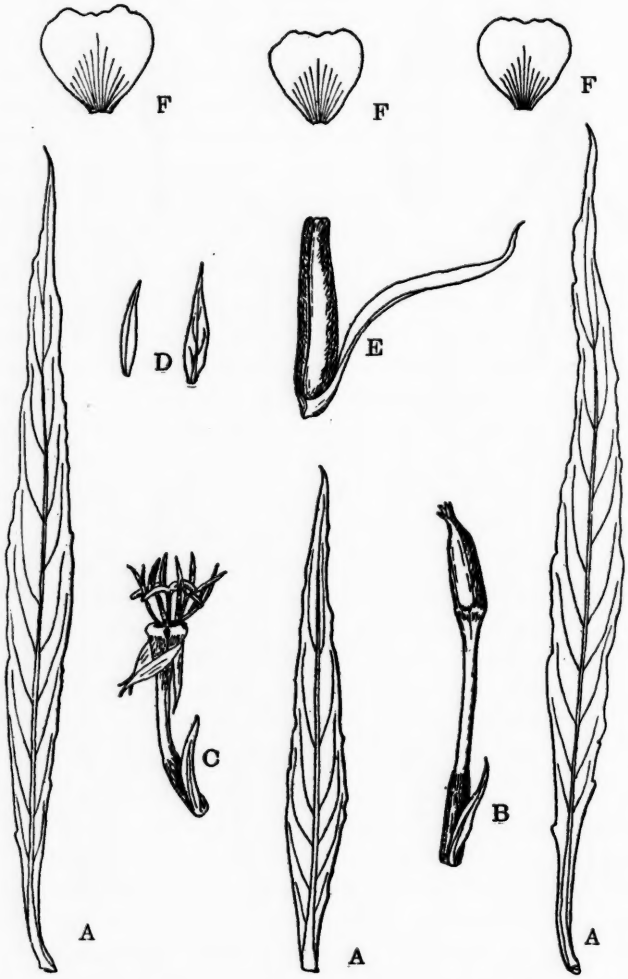
mutants. Another, *lata*, was imperfect and did not mature pollen, and hence was incapable of independent existence.

Others were perfect and vigorous both vegetatively and reproductively, and have been found capable of sustaining competition with the parental type. Of these *rubrinervis* grows more rapidly, germinates its seeds more quickly, and makes more numerous branches, and bids fair to be able to win out in a struggle with the parent in all of the phases of the struggle. *Gigas* is a more vigorous form than the parent, and both it and *rubrinervis* show a tendency to predominate when crossed with the parent. *Brevistylis* was found in the original location from which the mutating strain was taken, and as it has not appeared in any of the pedigreed cultures of the parent type in twenty years and still maintains itself in the original locality, it may be designated as a mutant which not only has arrived, but has survived under perfectly natural conditions. Recent cultures in the New York Botanical Garden from seeds of Lamarck's evening-primrose, sent from various parts of the world where the species is under cultivation, demonstrate that it is not alone the plants observed by De Vries at Hilversum are mutating, but the same derivatives are being given off in widely separated localities.

Purely-pedigreed strains of other species of evening-primroses have also been tested with the result that I am able to announce that three species are mutating, in lines of descent which came from wild plants one and two years ago. The common evening-primrose, *Oenothera biennis*, gives rise to one atypical individual in every two hundred seedlings, which during its entire development is continuously different from the parent in many features. It reproduces itself, after the manner of *scintillans*, appearing to be eversporting and giving many of the parental individuals in the purely fertilized seeds. Within the last two months a single rosette representing a new and unknown type has appeared in the seed-pans appearing to be a mutant which is found not more than once in twelve or fifteen



The Common Evening-primrose (*Oenothera biennis*), which is giving off one or more mutants. A, B, C, and D, leaves of the rosette. E, leaf from middle of flowering stem. F, bract. G, bud. H, flower with petals removed. I, petal. K, capsule.



A Mutant of the Common Evening-primrose. A, A, leaves of the rosette, and from the stem. B, flower-bud. C, flower with petals removed. D, bracts. E, capsule with bract in profile. F, petals.

thousand plants. The evening-primrose of the Adirondacks and northern New England, *Oenothera cruciata,* has been found to give atypic individuals conforming to a single type, which is also represented by specimens that have been collected in a wild state, so that here also, we have the survival of a species which is still arriving in a large proportion of the progeny. The great-flowered evening-primrose, *Oenothera grandiflora* of the southern states has been grown during two generations and it also is found to give derivatives, one or more of which appear to be already represented in the flora of the region.

One of the most interesting correlations to be made from a study of the results of these observations is to be found in the parallel mutations exhibited by the several species, in apparent contradiction of the principle of radiate variation and mutation (*allseitige Mutationen*). Among these are to be mentioned the origination of a form with cruciate flowers of the same general form as the *Oenothera cruciata* of the Adirondacks, from the species known as *O. biennis* in Holland, which is not identical with any species known to grow wild in America. The same species has also been seen to give off a mutant having the character corresponding to the mutant *nanella,* coming from *Lamarckiana,* according to De Vries. Then an evening-primrose of unknown identity has been found on Long Island by Dr. Shull, far removed from the locality inhabited by any other cruciate plant, and strongly suggestive of a mutation from *biennis* or some other form native to that locality. Such facts merely show that the forms borne by nearly related species lie well within the limits of the morphological possibilities in saltations and that they may be expected to be duplicated in other observations.

Scattered through the literature of botany and horticulture of the last century are scores of records of the

sudden appearance of sports and forms of the aspect of species which fully support all of the conclusions drawn from the observations on the evening-primroses. An examination of the facts easily brought together allows us to see that certain general principles in the organization of the plant, and in its behavior in these breaks or saltations in heredity may be made out.

The first and most important of these is one which was advanced by De Vries speculatively, before he began his experiments in heredity, namely that the plant is essentially a complex group of indivisible unit-characters. These unit-characters may not always be expressed, or recognizable in external anatomical characters, since they may be in a latent condition, or totally inactive, or external taxonomic characters may really consist of several elementary qualities, but these are not shown in any intermediate stage although they may be modified within the limits of fluctuating variability.

Any plant, supposedly, includes thousands of unit-characters, and as they are essentially qualities, or capacities, they do not usually coincide with the characters ordinarily used in taxonomic descriptions. As an illustration, the phases of geotropic sensibility of an organ may be considered as a unit-character. Thus a branch is either apogeotropic, directing its tip directly upward, or it may be diageotropic placing its axis in a horizontal plane, at right angles to the action of gravitation, or it may undergo a mutation and "weep" or direct its tips directly downward. In any case, however, it possesses one of these three forms of reaction. It does not follow, however, that all branches are actually in one of these three positions, for other forces to which it reacts may operate to place the axes in various planes, and the position of the branch may express just such concurrence of elementary characters as alluded to above, or indeed the geotropic unit-character may be latent,

and the organ may respond to other forces exclusively. Similar analyses must be used in the delineation of all unit-characters, and it is obvious, without further discussion, that we will never be able to uncover, either theoretically or actually, more than a few of these indivisible units. The forms and activities by which we recognize plants must by no means be taken to be simple in their constitution unless proved to be so, and the modification of a character in hybridization and otherwise must be taken as proof that it is not an elementary feature.

In tracing the development of species it is found practicable to designate them as retrogressive when a distinct unit-capacity, or unit-character is lost or, rather, becomes latent. Thus the loss of color must be of this kind, and also the loss of geotropic reaction, while the power of forming laciniate leaves when shown by mutants from a simple-leaved type would be estimated as a progressive mutation, as the group of characters concerned belong to a more highly organized type of organ. If a white-flowered species should give rise to a form with colored flowers it might well be taken as a degressive movement, or as a retracing of a step once lost, since all flowers were in all probability originally colored. Here again the actual test is hybridization, it being accepted that the retrogressive forms of organs are recessive or latent when crossed with the parental or nearly related types.

As to latency we can only say that strains of plants do carry capacities of one kind and another for many generations without these particular forms of unit-characters showing any activity. Thus a mutant which springs from a parental type, and shows a laciniate leaf instead of the ancestral leaf, must carry the latter form in a latent condition. The latent character may be awakened from time to time, or may be entirely and permanently inactive. The most easily analyzable examples of latency are seen in

hybrids. Thus when the blue-flowered *Veronica longifolia* was crossed with a white-flowered variety derived from it, the progeny was entirely and continuously blue-flowered except for occasional bud-sports. The white-flowered condition was here very evidently in a latent condition. In other instances white-flowered forms have appeared as a recessive, forming one-fourth of the progeny. If qualities have a cytological basis, and we may certainly assume that they do, then it is not easy to speculate intelligently on the probable condition of latent characters.

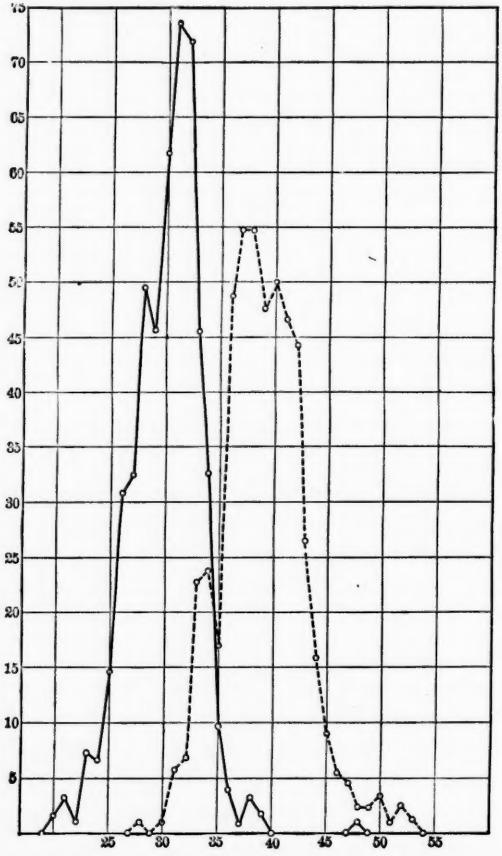
A very striking feature of mutants consists in the fact that the gross anatomical characters in which they diverge from the parent shows a much wider range of variability around its new norm, than does the homologous character in the parental type. Many observations bearing this interpretation have been on record for some time. Chas. Darwin in his *Origin of Species* noted that varieties varied more widely than the closely related species from which they were supposed to be derived, and many other observers have touched in one way or another upon the subject. Dr. Shull's recent researches upon this subject led him into the making of exact measurements, by which he also finds that the wide fluctuations of the mutant characters are accompanied by a lesser degree of correlation than prevails in the parental forms. Thus, for instance, the leaves of *rubrinervis* not only vary relatively more in width than those of the parental *Lamarckiana*, but the proportion between the width and length is not so constant as in the latter. Other organs of these and other species were subjected to exact measurements with similar results, while a large number of recent observations are known which justify the conclusions just stated. Of these the great variation of the length of the pistil in *brevistylis* in comparison, and the wide fluctuations in laciniate leaves are good examples. Many of the latter are known to be direct deriva-

tives of simple leaved ancestors, and are therefore capable of easy observation. With these facts in hand the possibility suggests itself that we might be able to distinguish between a structure recently arisen and one which was borne by a related species for thousands of years, but the exactness of such estimates is a matter of conjecture.

It seems to be taken for granted by a great number of workers interested in this subject, that species showing wide variations of the organs are the ones most likely to offer a high frequency of mutations, and my correspondents in various parts of the country are constantly calling attention to these forms under such a mistaken impression. As a matter of fact we may confidently expect that the species which show the greatest variation, or are eversporting, are the youngest.

Now having obtained the result just described we find ourselves face to face with one of the most interesting and difficult questions in heredity. If the newly arisen mutant forms are more widely variable than the older ones, how do they ultimately become narrowed? If the greater number of species originated by mutation, as we confidently believe they did, then they must have shown a much wider range of variability than they do at present; by the operation of what agency has variability been decreased, and correlations made more strict? At the present time I am compelled to say that I can not make an intelligent suggestion. Again if new characters vary widely at first, and lose this power, would it be possible to estimate the age of any given character of a species from the degree of variability? The author of the investigations just noted suggests that the best prospect for evidence of value upon this point might be obtained by a comparative statistical study of more recent types of structures in the foliar or reproductive organs, and of older forms that have come down from

the previous epoch. It certainly offers a most alluring field for research, and will doubtless soon receive attention.



Range of Variation of the Ratio of Width to Length of Leaves in the Parental *Lamarckiana* and One of Its Mutants. The dotted line shows that the range in the parent is 28-53, with a coefficient of variability of 9.53 ± 0.22 per cent., while in the mutant represented by the solid line, the range is 20-48, with a coefficient of variability of 10.30 ± 0.22 per cent.

Let us return to the question of the diversity of mutations which may ensue in any species. Theoretically a

plant might show discontinuous variation in almost any direction, but this departure of course does not exceed an amplitude determined by the morphological possibilities. With all of these features taken into consideration however, it is to be seen that it might be possible for any plant in a mutable condition to give rise to dozens, or perhaps

		X_2	−9	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8	9		
			50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140		
			55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145		
X_1																							
−12	17					1																1	
−11	18			1																		1	
−10	19																					0	
−9	20			1		1	3	1														6	
−8	21	1						1			1											3	
−7	22				1	2																3	
−6	23						2	2	7	2		1										14	
−5	24						1	2	6	2	1	2			1							15	
−4	25						3	2	6	6	3	5	2			1						28	
−3	26							3	5	6	11	3		1	2	1	1	1				34	
−2	27							2	15	11	7	9	6	3	3				1			57	
−1	28							3	4	10	8	16	5	9	1	2						58	
0	29							1	2	11	19	10	3	6	5	1						58	
1	30								1	13	12	10	16	10	8	1	2		1		1	75	
2	31				1					2	11	18	21	6	3	1	1	1	1			66	
3	32									1	8	17	20	8	7	3	3	1				68	
4	33										1	9	9	7	6	3	2					37	
5	34											1	6	15	5	7	1	2				37	
6	35										2	3	4	13	5	2	1					30	
7	36											1		1	3							5	
8	37										1	1		1	2	1	2					8	
9	38														2		1	2				5	
10	39														2	1	1					4	
11	40																1					1	
			1	2	2	4	9	17	46	64	85	106	92	80	55	24	16	7	3	0	1	614	

Correllation Table of Length and Width of Leaf in *O. rubrinervis*. $p = 0.6604 \pm 0.0119$.

scores of types simultaneously, while on the other hand it may originate but one aberrant form at a time. Lamarck's evening-primrose is producing a dozen, and the common species but one or possibly two, while the great-flowered species is throwing off three or four, as far as

our observations go. It will be profitable to analyze the consequences of the origination of diverse types simultaneously. In the case of *Oenothera grandiflora* it ranges from Kentucky southward to the Gulf over climatic and edaphic conditions of wide diversity. If it be assumed

			X_2	−9	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8	9	
				45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	
				50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	
X_1																							
+13	24			1					1														2
+12	25					1																	1
+11	26				1	1																	2
+10	27						1	1		1													3
−9	28						1	1		1		1											4
−8	29							1	2	3	2	1											9
−7	30							1	2	2	2	3											10
−6	31								1			2	2										5
−5	32									2	2	4		2									10
−4	33							1	1	6	3	4	4	3									22
−3	34								1	1	5	8	3	3	1								22
−2	35								1	2	4	9	6	1	4	3	1		1				32
−1	36							1	1	1	2	5	10	4	5	1		1					31
0	37								1		3	11	9	11	8	5	3	1					52
1	38								1		4	3	11	10	16	3	1						49
2	39											3	7	7	8	4	1	1					31
3	40											6	4	9	6	12	3	4					44
4	41												1	8	10	5	5	1	4	1			35
5	42												2	7	9	5	3	2	1		1		30
6	43										1			1	8	4	2	2			1		19
7	44														3		6		2		1		12
8	45														1	1	1	1	2				6
9	46																1						1
10	47																				1		1
11	48																	2			1		3
12	49														1					1			2
13	50													1									1
				1	1	2	2	6	12	19	28	60	59	67	80	43	27	15	10	2	1	4	439

Correllation Table of Length and Width of Leaf in *O. Lamarckiana*. $p = 0.7916 \pm 0.0090$.

that all of the mutants are being given off by the whole species then these must be thrown into the struggle for existence under conditions widely different. In some cases one mutant finds itself equipped to survive in the given

environment, and it does so in competition with the native flora, including the parental form. In a different part of the natural range of the parental form, a second mutant might have the advantage, and it alone of the entire brood would survive, while a third would be the best form for still another set of conditions. So well in accord with the facts do we find this assertion, that it is not hazardous to predict that when a final survey of the distribution of *Oenothera grandiflora* and its mutants is made, some such arrangement will be found. With this idea we must also concede that many of the mutants, so far as our experience goes, do not meet at all the conditions suitable for their existence, and these perish. In brief we have natural selection, not a selection *within* species, but a selection *among* species, by which certain ones are elected for survival and others doomed to destruction no matter how numerously, or how long they may be thrown off by the parental type.

As to the periodicity of mutations our information is not very extensive. Does a species, as it produces generation after generation in the course of centuries, arrive at a point where it begins to give off atypic individuals, and is this process continued for a time and then discontinued? We can only say that we find some species mutating and others not, we have not seen either the opening or closing of the mutative period in any species. This consideration is complicated however with that of the frequency of the mutants. Thus in Lamarck's evening-primrose five in every hundred plants are mutants, one in every two hundred of *biennis,* and it is conceivable that the atypic form might not occur more than once in a thousand, or once in ten thousand, or once in a million. These large numbers of plants of any species are not all in existence at any one time, and it might take years, or even decades to bring one mutant within the range of the possible number, in which case a false conception of the mutative period might be

gained. It is suggested therefore that the conception of *frequency of mutation* is the primary idea, although the action might become intensified in certain periods, of more or less definite limits.

Finally we may consider the causes inducing or affecting mutations. Mutants are found to be the most numerous under conditions most favorable to the growth and reproduction of the parental type, and this is also true of all anomalous structures. It is thus to be seen that mutations probably do not give rise to species most readily under the stress of unfavorable environment, or under any conditions which weaken the parental form, but when it is at a maximum of activity.

In the attempts to localize the changes in the cells, or in the chromosomes, which result in the formation of atypic individuals from seeds we are confident for theoretical reasons that these ensue previous to the reducing or qualitative divisions in the formation of the egg, or in the pollen mother-cells. Just what this change may be, we are unable to say, and shall probably know no more about it until the cytologist shall have given us some clue to the manner in which the separate qualities are represented in the chromosomes, in which the mutative changes must ensue. It is self-evident that in a mutation, some characters or qualities being borne along steadily from cell to cell in the divisions must be thrown into a latent condition, or perhaps totally lost, while simultaneously or separately, other qualities may be acquired by what actual operation we do not know. It seems fairly obvious, however, that these saltations arising from the non-uniform action of the chromosomes, must take place in response to some stimulus outside of the protoplast in which it actually occurs. This by no means supposes that the stimulation comes from climatic, or other environmental factors, but in all probability results from enzymatic or other action from neighboring

masses of cells. If this is so then we may hope to be able to duplicate the process in our cultures and call out a proportion of mutants at our will. The results of experiments now in progress seem to lend great favor to this assumption.

This view of the case is also favored by the facts offered by bud-sports, which have been designated as "vegetative mutants, although as shown above, all mutations are essentially of a vegetative character. In the simplest forms of these sports, lateral buds arising generally near the base of a shoot develop branches which diverge definitely from the characters of the main shoot, and which usually coincide with some known form, although this is not always the case. Sometimes entirely new forms arise in this manner as in the case of seed-mutants. During the present season I have been so fortunate as to have three notable examples of bud-sports in the experimental cultures. One of these was a basal branch of *Oenothera ammophila* which sported into the characters of *O. biennis,* and suggesting a possible hybrid ancestry with the latter species as one of the parents. A second case was one in which a seed-mutant of *O. biennis* gave a bud-sport which bore the characters of the ancestral type or the true *O. biennis.* A third case was one in which a plant of one of the numerous types embraced in a complex hybrid progeny bore a branch which sported in a branch which resembled a sister type. Other anatomical relations are found. Thus a bud-sport may embrace not only a branch, but a portion of the main stem, from which it arises, or in other cases it may include a section or longitudinal strip along one side of a branch even dividing a flower or fruit, while in other cases it may be represented by single flowers or fruits scattered indiscriminately through an inflorescence.

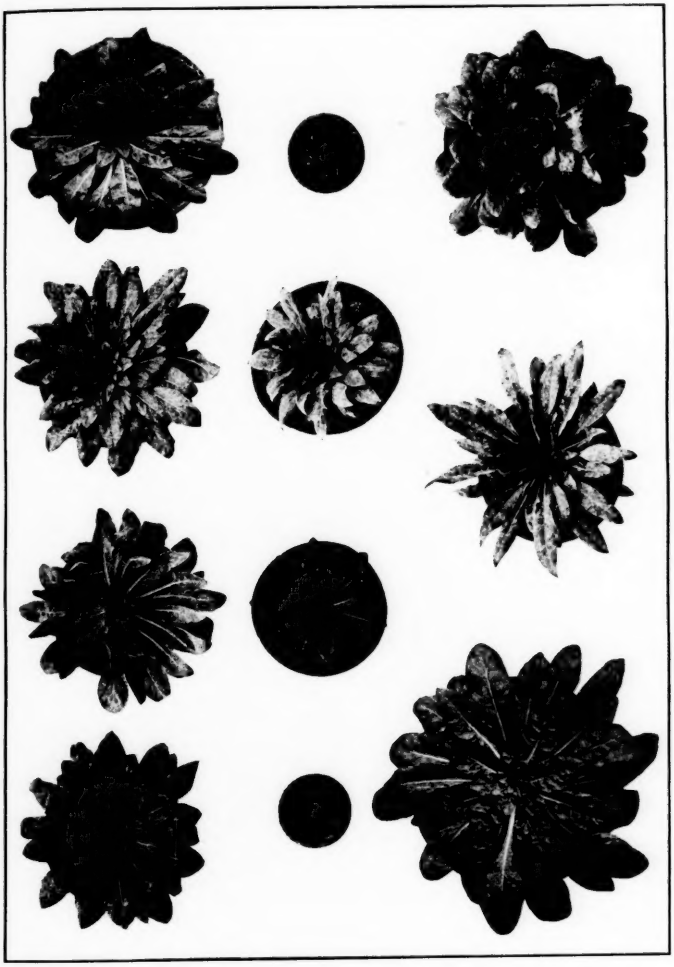
It is to be noted that in most if not all of the sectorial variations by which a part of a bud bears the divergent

characters, the change is a reversionary one, and the qualities that appear are really latent in the entire plant, and only need some stimulus to awaken them, or some agency to weaken the dominancy of the prevalent characters.

In the case of the appearance of characters not hitherto borne by the main stock, the case is not so clear, especially as we feel fairly certain that the saltations do ensue, in seed-mutants at least, in single cells. Here the theoretical side of the case seemed least supported by facts and I set about supplying the deficiency with what success you may presently judge.

Bud-sports are usually found on the basal portions of main stems of shoots, or in that part of the main axis, which often shows juvenile structures, so that we may say that these breaks in heredity are very frequent in the closing part of the embryonic period, although sometimes occurring much later. On the other hand, the changes which result in the formation of seed-mutants must ensue in the very closing stages of the sporophyte, and here is encountered a feature not recognized in bud-mutations, that of a fairly constant frequency, which in itself would suggest internal causes. It was determined however to attempt to induce or influence mutation in one of these periods. It was deemed impracticable to attempt this in the early life of the sporophyte and attention was given entirely to the stage immediately preceding the reducing divisions.

Omitting the detail of technique I may say that strong osmotic reagents, and weak solutions of stimulating mineral salts were injected into ovaries in such manner that unfertilized ovules were subjected to the action of the fluids, which killed many of them, but which gave the much desired results in a few. In *Lamarckiana* and *biennis* the frequency and character of the known mutants was unaffected, but in the progeny of the latter species was found a single individual constituting a type hitherto un-



Lamarck's Evening-primrose and Ten of Its Mutants. The dense rosette in the lower left-hand corner is *O. Lamarckiana*, the parental form. *O. gigas* is in the upper right-hand corner, and under it is *O. scintillans*. The irregular rosette above *O. Lamarckiana* is *O. albida*, and to the right of *albida* is *lata*. *O. oblonga* is in the upper left-hand corner.

known. This single aberrant individual might have been a mutant of low frequency, comparable to *gigas* derived from *Lamarckiana*, and its recurrence here might have been merely a matter of chance. However, in another species of evening-primrose, *Raimannia odorata*, a flower which belongs to a separate genus of the family, and is not known to be mutating, the treatment described resulted in a large number of aberrant individuals of a hitherto unknown type. Some of these, which show a shorter life-cycle than the parental form, and many anatomical divergencies, have been brought to bloom and to maturity, and the new form is obviously a potential species. In this experience, exemplified by specimens of the normal, parental forms, and aberrant mutants, I am able to offer you conclusive proof that agencies external to the cell may induce mutations, and consequently exert a profound influence on heredity. It would not be well to exaggerate the importance of this result, yet it is evident that the establishment of this fact marks a long step forward in the experimental study of inheritance and the origin of species.

A brief summary of the foregoing discussion may be made in the following generalizations:

1. Species may arise by hybridizations which result in fixed forms. A large number of forms known as species and recognized to be of such origin are known and a number of them have been duplicated in experimental cultures, some of which were made over a half century ago.

2. The mutation theory groups an enormous number of hitherto unexplainable facts, to which we are constantly adding in great volume, into a connected and meaningful whole, and best of all it brings the subject anew into a condition where it is amenable to experimental methods, in the laboratory, and experimental garden.

3. As a result of the theoretical conceptions offered us,

we have been able to make repeated observations of the general principles which govern breaks, or saltations in heredity, and to observe in what manner such mutations are connected with the origin of species.

4. Having ascertained at what time in the life-period of the individual mutations occur, I have been so fortunate as to secure results, demonstrating that mutations may be induced in a species not hitherto active in this respect, and that it is possible to call out new species by the intervention of external agents during the critical period.

5. Not less important than the foregoing is the unavoidable implication that breaks, saltations, or discontinuous action may be caused in inheritance by forces external to the protoplasts, and cells which are the true bearers of the hereditary characters.

Lastly it is of the greatest interest to note that in the effort to correlate the larger generalizations in the various departments of science in the concept of mutation we have hit upon a principle strongly favored by a modern system of mathematics, well exemplified by the spontaneous breaking-up and rearrangement of the complex atoms in radium, uranium and allied metals, and which has been recognized by Prof. George Darwin, the physicist, in the following words: "These considerations lead me to express a doubt whether biologists have been correct in looking for continuous transformation of species. Judging by analogy we should rather expect to find slight continuous changes occurring during a long period of time, followed by a somewhat sudden transformation into a new species, or by rapid extinction."

In the long-continued narrowing of the range of fluctuation in the various organs, coming to saltations, or direct origination of new forms, as the plant passes from generation to generation, we have as perfect a fulfilment of this motion as might be expected when an attempt is

made to interpret the action of the living by the properties of the non-living.

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MATHEMATICAL EMANCIPATIONS.

THE PASSING OF THE POINT AND THE NUMBER THREE: DIMENSIONALITY AND HYPERSPACE.

AMONG the splendid generalizations effected by modern mathematics, there is none more brilliant or more inspiring or more fruitful, and none more nearly commensurate with the limitless immensity of being itself, than that which produced the great concept variously designated by such equivalent terms as hyperspace, multidimensional space, *n*-space, *n*-fold or *n*-dimensional space, and space of *n* dimensions.

In science as in life the greatest truths are the simplest. Intelligibility is alike the first and the last demand of the understanding. Naturally, therefore, those scientific generalizations that have been accounted really great, such as the Newtonian law of gravitation, or the principle of the conservation of energy, or the all-conquering concept of cosmic evolution, are, all of them, distinguished by their simplicity and apprehensibility. To that rule the notion of hyperspace presents no exception. For its fair understanding, for a live sensibility to its manifold significance and quickening power, a long and severe mathematical apprenticeship, however helpful it would be, is not demanded in preparation, but only the serious attention of a mature intelligence reasonably inured by discipline to the exactions of abstract thought and the austerities of

the higher imagination. And it is to the reader having this general equipment, rather than to the professional mathematician as such, that the present communication is addressed.

To a clear understanding of what the mathematician means by hyperspace, it is in the first place necessary to conceive in its full generality the closely related notion of dimensionality and to be able to state precisely what is meant by saying that a given manifold has such and such a dimensionality, or such and such a number of dimensions, in a specified entity or element.

Discrimination, as the proverb rightly teaches, is the beginning of mind. The first psychic product of that initial psychic act is *numerical*: to discriminate is to produce *two,* the simplest possible example of multiplicity. The discovery, or better the invention, better still the production, best of all the creation, of multiplicity with its correlate of number, is, therefore, the most primitive achievement or manifestation of mind. Such creation is the immediate issue of intellection, nay, it *is* intellection, identical with its deed, and, without the possibility of the latter, the former itself were quite impossible. Accordingly it is not matter for surprise but is on the contrary a perfectly natural or even inevitable phenomenon that explanations of ultimate ideas and ultimate explanations in general should more and more avail themselves of analytic as distinguished from intuitional means and should tend more and more to assume arithmetic form. Depend upon it, the universe will never be really understood unless it may be sometime resolved into an ordered multiplicity and made to own itself an everlasting drama of the calculus.*

Let us, then, trust the arithmetic instinct as funda-

* In this connection I take pleasure in referring the reader to the somewhat naïve indeed but none the less very suggestive and stimulating article "The Warp of the World" by Mr. Newman Howard, *Hibbert Journal,* January, 1905.

mental and, for instruments of thought that shall not fail, repair at once to the domain of number. Every one who may reasonably aspire to a competent knowledge of the subject in hand is more or less familiar with the system of real numbers, composed of the positive and negative integers and fractions, such irrational numbers as $\sqrt{2}$ and π and countless hosts of similar numbers similarly definable. He may know that, for reasons which need not be given here, the system of real numbers is commonly described as the analytical continuum of first order. He knows, too, at any rate it is a fact which he will assume and readily appreciate, that the distance between any two points of a right line is exactly expressible by a number of the continuum; that, conversely, given any number, two points may be found whose distance apart is expressed by the numerical value of that number; that, therefore, it is possible to establish a unique and reciprocal, or one-to-one, correspondence between the real numbers and the points of a straight line, namely, by assuming some point of the line as a fixed point of reference or origin of distances, by agreeing that a distance shall be positive or negative according as it proceeds from the origin in this sense or in the other and by agreeing that a *point* and the *number* which by its magnitude reckoned in terms of a chosen finite unit however great or small serves to express the distance of the point from the origin and by its sign indicates on which side of the origin the point is situated, shall be a pair of *correspondents*. Accordingly, if the point P glides along the line, the corresponding number v will vary in such a way that to each position of the geometric there corresponds one value of the arithmetic element, and conversely. P represents v; and v, P. No two P's represent a same v; and no two v's, a same P. By virtue of the correlation thus established with the *analytical* continuum, we may describe the line as a simple

or one-fold *geometric* continuum, namely, of points. The like may in general be said, and for the same reason, of any curve whatever, but we select the straight line as being the simplest, for in matters fundamental we should prefer clearness to riches of illustration, in the faith that, if first we seek the former, the latter shall in due course be added unto it. The straight line, when it is regarded as the domain of geometric operation, as the region or room containing the configurations or sets of elements with which we deal, is and is called a *space;* and this space, viewed as the manifold or assemblage of its points, is said to be *one*-dimensional for the reason that, as we have seen, in order to determine the position of a point in it, in order, i. e., to pick out or distinguish a point from all the other points of the manifold, it is necessary and sufficient to know one fact about the point, as, e. g., its distance from an assumed point of reference. In other words, the line is called a one-dimensional space of points because in that space the point has one and but one degree of freedom or, what is tantamount, because the position of the point depends upon the value of a single v, known as its coordinate.

Herewith is immediately suggested the generic concept of dimensionality: *if an assemblage of elements of any given kind whatsoever, geometric or analytic or neither, as points, lines, circles, triangles, numbers, notions, sentiments, hues, tones, be such that, in order to distinguish every element of the assemblage from all the others, it is necessary and sufficient to know exactly n independent facts about the element, then the assemblage is said to be n-dimensional in the elements of the given kind.* It appears, therefore,that the notion of dimensionality is by no means exclusively associated with that of space but on the contrary may often be attached to the far more generic concept of assemblage, aggregate or manifold. For ex-

ample, duration, the total aggregate of time-points, or instants, is a simple or one-fold assemblage. On the other hand, the assemblage of colors is three-dimensional as is also that of musical notes, for in the former case, as shown by Clerk Maxwell, Thomas Young and others, every color is composable as a definite mixture of three primary ones and so depends upon three independent variables or co-ordinates expressing the amounts of the fundamental components. And in the latter case a similar scheme obtains, one note being distinguishable from all others when and only when the three general marks, pitch, length, and loudness, are each of them specified. In passing it seems worth while to point out the possibility of appropriating the name *soul* to signify the *manifold* of all *possible* psychic *experiences,* in which event the term would signify an assemblage of probably *infinite* dimensionality, and the assemblage would be continuous, too, if Oswald* be right in his contention that every manifold of experience possesses the character of continuity. That contention, however, if the much abused term continuity be allowed to have its single precise definitely seizable scientific meaning, is far less easy to make good than that eminent chemist and courageous philosopher seems to think.

Returning to the concept of space, an *n*-fold *assemblage* will be an *n*-dimensional *space* if the elements of the assemblage are geometric entities of any given kind. We have seen that the straight line is a *one*-dimensional space of *points.* But in studying the right line conceived as a space, we are not compelled to employ the point as element. Instead we may choose to assume as element the point *pair* or *triplet* or *quatrain,* and so on. The line would then be for our thought primarily a space, not of points, but of point pairs or triplets and so on, and it would accordingly be strictly a space of *two dimensions* or of *three,*

* Cf. his *Natur-Philosophie.*

and so on; for, obviously, to distinguish say a point pair from all other such pairs we should have to know *two* independent facts about the pair, one of which would determine one of the points, and the other, the other. The pair would have two degrees of freedom in the line, its determination would depend upon two independent variables as v_1 and v_2 referred to two (in general different) points of reference. It should be signalized that a pair of points is not merely two points, nor a triplet merely three. *Order* is essential, and two points will furnish two distinct pairs, and three points will furnish six pairs or six triplets; on the other hand, three pairs or three triplets may contain but three points. On its arithmetic side the shield presents a precisely parallel doctrine. The simple analytical continuum composed of the real numbers immediately loses its simplicity and assumes the character of a 2- or 3- . . . or n-fold analytical continuum if, instead of thinking of its individual numbers, we view it as an aggregate of number pairs or triplets or, in general, as the totality of ordered systems of n numbers each.

In the light of the preceding paragraph it is seen that the dimensionality of a given space is not unique but depends upon the choice of geometric entity for primary or generating element. A space being given, its dimensionality is not therewith determined but depends upon the will of the investigator, who by a proper choice of generating element may endow the space with any dimensionality he pleases. That fact is of cardinal significance alike for science and for philosophy. I reserve for a little while its further consideration in order to present at once a kind of complementary fact of equal interest and of scarcely less importance. It is that two spaces which in every other respect are essentially unlike, thoroughly disparate, may, by suitable choice of generating elements, be made to assume equal dimensionalities. Consider, for example,

the totality of lines contained in a same plane and containing a point in common. Such a totality, called a *pencil*, of lines is a simple geometric continuum, namely, of lines. It is, then, and may be called, a *one*-dimensional space of *lines* just as the line or *range* of points is a one-dimensional space of points. The two spaces are equally rich in their respective elements. And if, following Desargues and his successors, we adjoin to the points of the range a so-called "ideal" point or point at infinity, thus rendering the range like the pencil, *closed*, it becomes obvious that two intelligences, adapted and confined respectively to the two simple spaces in question, would enjoy equal freedom; their analytical experiences would be identical, and their geometries, though absolutely disparate in kind, would be equally rich in content. Just as the range-dweller would discover that the dimensionality of his space is *two* in *point pairs, three* in *triplets,* and so on, so the pencil-inhabitant would find his space to be of dimensionality *two* in *line pairs, three* in *triplets,* and so on without end. It was indicated above that *any* line, straight or *curved,* is a *one*-dimensional space of *points*. In that connection it remains to say that, speaking generally, *any curve,* literally and strictly conceived as the assemblage of its (tangent) lines and so including the point or pencil as a special case, is also a *one*-dimensionel space of *lines*. It is, moreover, obvious that the foregoing considerations respecting the range of points and the pencil of lines are, *mutatis mutandis,* equally valid for any one of an infinite variety of other analogous spaces, as, e. g., the *axal pencil,* a one-fold space of planes, consisting of the totality of planes having a line in common.

If perchance some reader should feel an ungrateful sense of impropriety in our use of the term *space* to signify such common geometric aggregates as we have been considering, I gladly own that his state of mind is a per-

fectly natural one. But it is, besides and on that account, a source of real encouragement. Dictional sensibility is a hopeful sign, being conclusive evidence of life, and, while there is life, there remains the possibility and therewith the hope of readjustment. In the present case, I venture to assure the reader, on grounds both of personal experience and of the experience of others, that whatever sense he may have of injury received will speedily disappear in the further course of his meditations. Only, let him not be impatient. Larger meanings must have time to grow; the smaller ones, those that are most natural and most provincial, being also the most persistent. In the process of clarification, expansion and readjustment, his fine old word, space, early come into his life and gradually stained through and through with the refracted partial lights and multi-colored prejudices of his youth, is not to be robbed of its proper charms nor to be shorn of its proper significance. More than it will lose of mystery, it shall gain of meaning. Of this last it has hitherto had for him but little that was of scientific value, but little that was not vague and elusive and ultimately unseizable. That was because the word stood for something absolutely *sui generis, i. e., for a genus neither including species nor being itself included in a class.* But now, on the other hand, both of these negatives are henceforth to be denied, and the hitherto baffling term, perfect symbol of the unthinkable, always promising and never presenting definable content, immediately assumes the characteristic twofold aspect of a genuine concept, being at once included as member of a higher class, the more generic class of manifolds, and including within itself an endless variety of individuals, an infinitude of species of space.

Of these species, the next, in order of simplicity, to those above considered, is the plane. To distinguish a point of a plane from all its other points, it is necessary

and sufficient to know *two* independent facts about its position, as, e. g., its distances from two assumed lines of reference, most conveniently taken at right angles. Viewed as the *ensemble* of its points, the plane is, therefore, a space of *two* dimensions. In that space, the point enjoys a freedom exactly twice that of a point in a range or of a line in a pencil, and exactly equal to that of a *pair* of points or of lines in the last-mentioned spaces. On the other hand, if the point *pair* be taken as element of the plane, the latter becomes a space of *four* dimensions.

What if the *line* be taken as generating element? It is obvious that the plane is equally rich in pencils and in ranges. It contains as many lines as points, neither more nor less. Two points determine a line; two lines, a point; if the lines be parallel, their common point is a Desarguesian, a point at infinity. We should therefore expect to find that in a plane the position of a line depends upon two and but two independent variables. And the expectation is realized, as it is easy to see. For if the variables be taken to represent (say) distances measured from chosen points along two lines of reference, it is immediately evident that a given pair of values of the variables determines a line uniquely and that, conversely, a given line uniquely determines such a pair. The plane is, therefore, a *two*-dimensional space of *lines* as well as of points. In line *pairs*, as in point pairs, its dimensionality is *four*. We may suppose the space in question to be inhabited by two sorts of individuals, one of them capable of thinking in terms of points but not of lines, the other in terms of lines but not of points. Each would find his space bi-dimensional. They would enjoy precisely the same analytical experience. Between their geometries there would subsist a fact-to-fact *correspondence* but not the slightest *resemblance*. For example, the circle would be for the former individual a certain assemblage of points but devoid of

tangent lines, and, for the latter, a corresponding assemblage of (tangent) lines but devoid of contact points.

Passing from the plane to a curved surface, to a sphere, for example, a little reflection suffices to show that the latter may be conceived in a thousand and one ways, most simply as the ensemble of its points *or* of its (tangent) planes *or* of its (tangent) lines. These various concepts are logically equivalent and in themselves are equally intelligible. And if to us they do not seem to be also equally *good,* that is doubtless because we are but little traveled in the great domain of Reason and therefore naturally prefer our familiar customs and provincial points of view to others that are strange. At all events, it is certain that on purely *rational* grounds, none of the concepts in question is to be preferred, while, from preference based on *other* grounds, it is the office alike of science and of philosophy to provide the means of emancipation. Let us, then, detach ourselves from the vulgar point of view and for a moment contemplate the three concepts as coordinate indeed but independent concepts of surface. And for the sake of simplicity, we may think of a sphere.* Suppose it placed upon a plane and imagine its highest point, which we may call the pole, joined by straight lines to all the points of the plane. Each line pierces the sphere in a second point. It is plain that thus a one-to-one correspondence is set up between the points of the sphere and those of the plane, except that the pole corresponds at once to all the Desarguesian points of the plane—an exception, however, which is here of no importance. The plane and the sphere are, then, equally rich in points. Accordingly, the sphere conceived as a plenum or locus of *points* is a space of *two* dimensions. In that space the point has two degrees of freedom. Its position depends upon two independent vari-

* The term is here employed as in the higher geometry to denote, not a solid, but a surface.

ables, as latitude and longitude. But we may conceive the surface quite otherwise: at each of its points there is a (tangent) plane, and now, *disregarding points,* we may think only of the assemblage of those planes. These together constitue a sphere, not, however, as a locus of points, but as an envelope (as it is called) of planes. And what shall we say of the surface as thus conceived? The answer obviously is that it is a *two*-dimensional space of *planes,* admitting of a geometry quite as rich and as definite as is the theory of any other space of equal dimensionality. In each of the planes there is a pencil of lines of which each is tangent to the sphere. Thus we are led to a third conception of our surface. We have merely to disregard both points and planes and confine our attention to the assemblage of lines. The vision which thus arises is that of a *three*-dimensional space of *lines.* In pencils, its dimensionality is two. In this space the pencil has two and the line three degrees of freedom.

But let us return to the plane. We have seen that at the geometrician's bidding it plays the rôle of a twofold space either in points or in lines. It is natural to ask whether it may be conceived as a space of *three* dimensions, like the sphere in its third conception. The answer is affirmative: it may be so conceived, and that in an infinity of ways. Of these the simplest is to assume the *circle* as primary or generating element. Of circles the plane contains a threefold infinity, an infinity of infinities of infinity: It is a circle continuum of third order. To distinguish any one of its circles from all the rest, three independent data, two for position and one for size, are necessary and sufficient. In the plane the circle has three degrees of freedom, its determination depends upon three independent variables. The plane is, accordingly, a tridimensional space of *circles.* In *parabolas* its dimensionality is *four;* in *conics, five;* and so on *without limit.*

Before turning to space, ordinarily so-called, it seems worth while to indicate another geometric continuum which, although it presents no *likeness* whatever to the plane, nevertheless *matches* it perfectly in every conceptual aspect. The reference is to the *sheaf*, or *bundle*, of *lines*, i. e., the totality of lines having a point in common. The point is to be disregarded and the lines viewed as non-decomposable entities, like points in a line or a plane regarded as an assemblage of points. Thus conceived, the *sheaf* is literally a *space*, namely, of *lines*. It is, in the vulgar sense of the term, just as *big*, occupies precisely as much room, nay indeed the same room, as the space in which we live. The sheaf as a space is *two*-dimensional in *lines*, like the *plane* in *points; two*-dimensional in *pencils*, like the *plane* in *lines; four*-dimensional in *line* or *pencil pairs*, like the *plane* in *point* or *line pairs; three*-dimensional in ordinary *cones*, like the *plane* in *circles;* and so on and on.

In the light of the foregoing considerations, any hitherto uninitiated reader will probably suspect that ordinary space is *not*, as it is commonly supposed and said to be, an *inherently* and *uniquely three*-dimensional affair. His suspicion is completely justified by fact. The simple traditional affirmation of tri-dimensionality is devoid of definite meaning. It is unconsciously elliptic, requiring for its completion and precision the specification of an appropriate geometric entity for generating element. Merely to say that space is tri-dimensional because a solid, e. g., a plank, has length, breadth and thickness, is too crude for scientific purposes. Moreover, it betrays, quite unwittingly indeed as we shall see, an exceedingly meager point of view. Not only does it assume the *point* as element but it does so tacitly because unconsciously, as if the point were not merely *an* but *the* element of ordinary space. *An* element the point may obviously be taken to be, and in

that element ordinary space is indeed tri-dimensional, for the position of a point at once determines and is determined by three independent data, as its distances from three assumed mutually perpendicular planes of reference. It must be admitted, too, that the point does, in a sense, recommend itself as the element *par excellence,* at least for practical purposes. For example, we prefer to do our drawing with the point of a pencil to doing it with a straight edge. But that is a matter of physical as distinguished from rational convenience. Preference for the point has, then, a cause: in the order of evolution, practical man precedes man rational and determines for the latter his initial choices. Causes, however, are extra-logical things, and the preference in question, though it has indeed a cause, has no reason. Accordingly, when in these modern times, the geometrician became clearly conscious that he was in fact and had been from time immemorial employing the point as element and that it was this use that lent to space its traditional triplicity of dimensions, he did not fail to perceive almost immediately the logically equal possibility of adopting at will for primary element any one of an infinite variety of other geometric entities and so the possibility of rationally endowing ordinary space with any prescribed dimensionality whatever.

Thus, for example, the plane is no less available for generating element than is the point. The plane is logically and intuitionally just as simple, for, if from force of habit, we are tempted to analyze the plane into an assemblage of points, the point is in its turn equally conceivable as or analyzable into an assemblage of planes, the sheaf of planes containing the point. We may, then, regard our space as primarily a plenum of planes. To determine a plane requires three and but three independent data, as, say, the distances to it measured along three chosen lines from chosen points upon them. It follows that ordinary

space is *three*-dimensional in *planes* as well as in points. But now if (with Plücker) we think of the *line* as element, we shall find that our space has *four* dimensions. That fact may be seen in various ways, most easily perhaps as follows. A line is determined by any two of its points. Every line pierces every plane. By joining the points of one plane to all the points of another, all the lines of space are obtained. To determine a line it is, then, enough to determine two of its points, one in the one plane and one in the other. For each of these determinations, two data, as before explained, are necessary and sufficient. The position of the line is thus seen to depend upon four independent variables, and the four-dimensionality of our space in lines is obvious. Again, we may (with Lie) view our space as an assemblage of its *spheres*. To distinguish a sphere from all other spheres, we need to know four and but four independent facts about it, as, say, three that shall determine its center and one its size. Hence our space is *four*-dimensional also in *spheres*. In *circles* its dimensionality is *six;* in *surfaces* of *second order* (those that are pierced by a straight line in two points), *nine*; and so on *ad infinitum*.

Doubtless the reader is prepared to say that, if the foregoing account of hyperspace be correct, the notion is after all a very simple one. Let him be assured, the account is correct and his judgment is just: the notion is simple. That property, as said in the beginning, is indeed one of its merits. As presented the concept is entirely free from mystery. To seize upon it, it is unnecessary to pass the bounds of the visible universe or to transcend the limits of intuition. Its realization is found even in the line, in the pencil, in the plane, in the sheaf, here, there and yonder, everywhere, in fact. The account, however, though quite correct, is not yet complete. The term hyperspace has yet another meaning and yet in strictness not

another, as we shall see. It will be noticed that among the foregoing examples of hyperspace, none is presented of dimensionality *exceeding three* in *points.* It is precisely this variety of hyperspace that the term is commonly employed to signify, particularly in popular enquiry and philosophical speculation. And it is this variety, too, that just because it baffles the ordinary visual imagination, proves to be, for the non-mathematician at any rate, at once so tantalizing, so mysterious and so fascinating.

It remains, then, to ask, what is meant by a hyperspace of *points?* How is the notion formed and what is its motivity and use? The path of enquiry is a familiar one and is free from logical difficulty. Granted that a one-to-one correspondence can be established between the real numbers and the points of a right line, so that the geometric serve to represent the arithmetic elements; granted that all (ordered) pairs of numbers are similarly representable by the points of a plane, and all (ordered) triplets by the points of ordinary space; the suggestion then naturally presents itself that, whether there really is or not, there *ought* to be a *space* whose *points* would serve to *represent,* as in the preceding cases, *all ordered systems* of *values* of *n independent variables;* and especially to an analyst with a strong geometric predilection, to one who is a born *Vorstellender,* for whom analytic abstractions naturally tend to take on figure and assume the exterior forms of sense, that suggestion comes with a force which he alone perhaps can fully appreciate. And what does he do? Not finding the desiderated hyperspace present to his *vision* or *intuition* or *visual imagination,* he *posits* it, or if you prefer, he *creates* it, *in thought.* The concept of hyperspace of points is thus seen to be off-spring of Arithmetic and Geometry. It is legitimate fruit of the indissoluble union of the fundamental sciences.

Does such hyperspace exist? It does exist genuinely.

If not for intuition, it exists for conception; if not for imagination, it exists for thought; if not for sense, it exists for reason; if not for matter, it exists for mind. These if's are *if*'s in fact. The question of imaginability is really a question. We shall return to it presently.

The concept of hyperspace of points is generable in various other ways. Of all ways the following is perhaps the best because of its appeal at every stage to intuition. Let there be two points and grant that these determine a *line,* point-space of *one* dimension. Next posit a point *outside* of this line and suppose it joined by lines to all the points of the given line. The points of the joining lines together constitute a *plane,* point-space of *two* dimensions. Next posit a point outside of this plane and suppose it joined by lines to all the points of the plane. The points of all the joining lines together constitute an ordinary *space,* point-space of *three* dimensions. The clue being now familiar to our hand, let us boldly pursue the opened course. Let us overleap the limits of common imagination, transcend ordinary intuition as being at best but a non-essential auxiliary, and in *thought* posit an extra point that, *for thought* at all events, shall be *outside* the space last generated. Suppose that point joined by lines to all the points of the given space. The points of the joining lines together constitute a point-space of *four* dimensions. The process here applied is perfectly clear and obviously admits of endless repetition.

Moreover, the process is equally available for generating hyperspaces of other elements than points. For example, let there be two intersecting lines and grant that these determine a *pencil,* line-space of *one* dimension. Next posit a line (through the vertex) *outside* of the given pencil and suppose it joined by pencils to all the lines of the given pencil. The lines of the joining pencils together constitute a *sheaf,* line-space of *two* dimensions. Next

posit a line (through the vertex) *outside* of the sheaf and suppose it joined by pencils to all the lines of the sheaf. The lines of the joining pencils constitute a *hypersheaf*, line-space of *three* dimensions. The next step plainly leads to a line-space of *four* dimensions; and so on *ad infinitum*.

And now as to the question of imaginability. Is it possible to intuit configurations in a hyperspace of points? Let it be understood at the outset that that is not in any sense a mathematical question, and mathematics as such is quite indifferent to whatever answer it may finally receive. Neither is the question primarily a question of philosophy. It is first of all a psychological question. Mathematicians, however, and philosophers are also men and they may claim an equal interest perhaps with others in the profounder questions concerning the potentialities of our common humanity. The question, as stated, undoubtedly admits of affirmative answer. For the lower spaces, with which the imagination *is* familiar, exist *in* the higher, as the line in the plane, and the plane in ordinary space. But that is not what the question means. It means to ask whether it is possible to imagine hyper-configurations of points, i. e., point-configurations that are not wholly contained in a point-space (like our own) of three dimensions. It is impossible to answer with absolute confidence. One reason is that the term imagination still awaits precision of definition. Undoubtedly just as three-dimensional figures may be represented in a plane, so four-dimensional figures may be represented in space. That, however, is hardly what is meant by imagining them. On the other hand, a four-dimensional figure may be rotated and translated in such a way that all of its parts come one after another into the threefold domain of the ordinary intuition. Again, *the structure of a fourfold figure, every minutest detail of its anatomy, can be traced out by analogy with its three-dimensional analogue.* Now in such

processes, repetition yields skill, and so they come ultimately to require only amounts of *energy* and of *time* that are quite *inappreciable*. Such skill once attained, the *parts* of a familiar *fourfold configuration* may be made to pass before the eye of intuition in such *swift* and *effortless* succession that the configuration *seems present as a whole in a single instant*. If the *process* and *result* are not, properly speaking, *fourfold imagination* and *fourfold image*, it remains for the psychologist to indicate what is lacking.

Certainly there is naught of absurdity in supposing that under suitable stimulation the human mind may in course of time even speedily develop a spatial intuition of four or more dimensions. At present, as the psychologists inform us and as every teacher of geometry discovers independently, the spacial imagination, in case of very many persons, comes distinctly short of being strictly even tridimensional. On the contrary, it is flat. It is not every one, even among scholars, that with eyes closed can readily form a visual image of the *whole* of a simple *solid* like a sphere, enveloping it completely with bent beholding rays of psychic light. In such defect of imagination, however, there is nothing to astonish. In the first place, man as a race is only a child. He has been on the globe but a little while, long indeed compared with the fleeting evanescents that constitute the most of common life, but very short, the merest fraction of a second, in the infinite stretch of time. In the second place, circumstances have not, in general, favored the development of his higher potentialities. His chief occupation has been the destruction and evasion of his enemies, contention for mere existence against hostile environment. Painful necessity, then, has been the mother of his inventions. That, and not the vitalizing joy of self-realization, has for the most part determined the selection of the fashion of his faculties.

But it would be foolish to believe that these have assumed their final form or attained the limits of their potential development. The imperious rule of necessity will relax. It will never pass quite away but it will relax. It is relaxing. It has relaxed appreciably. The intellect of man will be correspondingly quickened. More and more will joy in its activity determine its modes and forms. The hyper-dimensional worlds that man's reason has already created, his imagination may yet be able to depict and illuminate.

It remains to ask, finally, what purpose the concept of hyperspace subserves. Reply, partly explicit but chiefly implicit, is not, I trust, entirely wanting in what has been already said. Motivity, at all events, and *raison d'être* are not far to seek. On the one hand, the great generalization has made it possible to enrich, quicken and beautify analysis with the terse, sensuous, artistic, stimulating language of geometry. On the other hand, the hyperspaces are in themselves immeasurably interesting and inexhaustibly rich fields of research. Not only does the geometrician find light in them for the illumination of many otherwise dark and undiscovered properties of the ordinary spaces of intuition, but he also discovers there wondrous structures quite unknown to ordinary space. These he examines. He handles them with the delicate instruments of his analysis. He beholds them with the eye of the understanding and delights in the presence of their supersensuous beauty.

It is by creation of hyperspaces that the rational spirit secures release from limitation. In them it lives ever joyously, sustained by an unfailing sense of infinite freedom.

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FECHNER'S VIEW OF LIFE AFTER DEATH.

GUSTAV Theodor Fechner is best known as the formulator of the fundamental psychophysical law bearing his name, which is expressed in an equation as follows: $I = C \log S$, in which I is intensity of sensation, S the stimulus and C a constant peculiar to each individual that may vary in the several senses. It is an application of Weber's law for psychical measurements, and is an attempt at determining the proportion between the stimulus and its correspondent sensation.

Fechner was a professor of physics, but he took great interest in psychology and by combining the two sciences became one of the founders of a new science called "psychophysics," based upon the obvious interrelation between sensation and nerve-activity. While he did much creditable work in the line of exact psychology, he devoted himself with preference to those problems of the soul which touch upon its religious and moral life and its fate after death. His little book *On Life After Death* is his most important publication in this line.

Although I sympathize to a great extent with Professor Fechner's views, I cannot agree with him in some of his most important details which rise from a conception of the soul that might be regarded as spiritualistic. Fechner was not a spiritualist and even exhibited a decided dislike for spiritualistic séances, but he believed in the spirituality of the soul, and especially in a personal and conscious

immortality. Yet his views are peculiar and differ greatly from the current belief in that they are based upon certain scientific considerations. It seems worth while to study them, for they contain much that is good and true; and though one of his most essential arguments is wanting in logical conclusiveness, it is highly desirable that his conception be better known and more generally appreciated.

We propose here to sketch Fechner's view of the soul and point out where we part company with him.

Professor Fechner divides human life into first, the period before birth; then, life from birth to death; and thirdly, existence after death. He makes use of some ingenious analogies between the relation of man's prenatal condition in his mother's womb to his actual life, and of actual life to his fate after death. But in my opinion the division is not well chosen, and so the analogies fall to the ground. Man's life in this world does not begin with birth but with conception; and we ought to consider as his pre-existence, not his fœtal life, but all the causes which in their co-operation lead to the rise of his personality as a concrete being endowed with reason and pursuing purposes according to his individual character. This individual existence is dissolved in death, and the great question with mankind has ever been, If a man die, shall he live again?*

It may be wise to insist first of all upon the reality of soul-life. Not only are our sentiments actual events (indeed they are the stuff that life is made of, and are the bottom facts of all experience) but they are even more real than material objects; for after all they are un-

*The problem of man's fate after death ought to be determined by an investigation of the conditions that caused his present life, and in this sense the author has tried to solve the problem in a pamphlet entitled *Whence and Whither?* (Chicago, The Open Court Publishing Company, 1900). As it is not necessary to repeat the arguments here, the reader is referred to it for questions of detail.

equivocal and primary, while the existence of matter is a mere inference, a deduction made from sensations, since matter itself is but a name that signifies certain constants in the complex causes of our sensations. Yet, since sensations are fleeting and depend upon very complex conditions, it is claimed that they are unsubstantial and transient phenomena. While this may be true, we must point out that they are none the less actual for they are efficient; they leave traces, and so each mental state continues to live in its effect. The continuity of a person depends upon the preservation of his sentiments, as they are registered in the nervous tissue, the psychological aspect of which is called memory.

Soul-life may be fleeting, but there is a stability in its phenomena that is simply an application of the general law of cause and effect which makes every fact that ever happened an indestructible factor in the constitution of the world. Whatever has been done leaves an indelible trace constituting the past in the development of the world; and in its entire makeup the act is irrevocable, and is at the same time the determinant of the present and through the present, of the future. Accordingly, it is obvious that man's soul being a part of the web and woof of the universe will stay forever, and if a man dies we can state at once that his life will have its influence upon the future—it will be incorporated in the constitution of the universe, at least as much as any other event.

But man's life is preserved in a higher degree in the lives of future generations. Every single individual is, physically considered, but the continued growth of his parents, and through example as well as education he imbibes the quintessence of all the spiritual wealth of former generations. Every soul is made a trysting place of former souls. The thoughts of others continue in us. Every mind is a great republic of ideas, of impulses, of senti-

ments. It is a congregation of spirits who constitute a community in which they co-operate or antagonize each other; and in the measure that we live, and work, and are up and doing, we communicate our modes of thinking to others. With every sentence that we speak, or write, or have printed, and with every example which we set in our acts, we scatter spiritual seeds. Each soul migrates into others and takes root in a new soil, where it sprouts, continuing its own life under new circumstances, taking a new start, and yet remaining the selfsame tendency, the selfsame aspiration, the selfsame spirit.

It is apparent that the individual is not a true unit. We might with greater truth consider the ideas which take hold of the soul as units—as citizens which take up their abode in one mind and continue their existence in others. On the other hand we might also with greater truth consider the whole of mankind as a unit while man's individual existence is merely a middle link between the two, a device by which they are temporarily realized in flesh and blood in order to give them tangible existence.

Fechner's book is refreshing because his exposition of soul-life after death insists vigorously on the reality of this spiritual life which plays such an essential part in the constitution of our individual existence.

The memory of the dead is not a mere abstraction or sham, but constitutes the true presence of the souls of our beloved ones. They are here within us—nearer to us than our heart-beat; for their very souls, their thoughts, the examples which they set us, and their characters, constitute a portion of our own being. In the same way we, too, lead a spiritual life in the thoughts of others, our friends and also our foes, which will continue even when we are gone, and thus, though man must die, his soul will live.

Some details of Fechner's conception of life after death

are graphically described and unequivocally true. But he is misguided by his allegories when comparing man's life to his prenatal existence in his mother's womb, and thus the comparisons are frequently inappropriate. Our pre-existence is of a spiritual kind, and is not individual though it is definite and concrete. It is scattered over the whole realm of earthly conditions, and incorporates in an unbroken line our entire direct ancestry since the beginning of life on earth which is represented in innumerable simultaneous individuals. It is obvious that the constituents of one soul have been in many different individuals at once, and so also after death the soul of the departed will reappear not only once, but in many personalities whose destinies it will more or less determine.

But people clamor for a conscious immortality, saying that if they do not know that they are the same personalities they will not care for living after death. I would answer first of all that as much as in our present souls the the spirits of the past become conscious again, so our ideals and aspirations will be lighted up by the consciousness of future generations. If it be claimed that the consciousness of others is not our consciousness, I would answer that their very demand is based upon a gross materialistic conception of sameness; for even in our present existence every day's consciousness is in its way different from every other; and its successive phases can as little, and yet at the same time, as much, be regarded as the same, as the consciousness of successive generations. The whole problem is a matter of continuity, and the continuity of consciousness in one individual is as much broken up by sleep into successive days, as in the whole of mankind it is broken up into separate individuals. Every individual is as much a continuation of former individuals as one day in our life is the continuation of former days. In both cases continuity is complete, and the disruption

by sleep or by death touches merely the surface and does not disturb the integrity of the process of growth, evolution, historic development, or whatever we may call the cosmic background of our immortality.

The point where I would differ from Professor Fechner is when he attributes an objective existence to spirits as if they were concrete beings endowed with a consciousness of their own, and being able to act as if they were still individual beings like ourselves in what he describes as the second or earthly stage of being.

I do not deny that every soul remains a unit of its own. Great personalities, such as Christ, Shakespeare, Goethe, Washington, impress themselves on coming generations; and though they live in many, they retain their individuality and typical character. What is true of these great men is true in a minor degree of every one of us, but the unit of our individuality is not atomistic. It is not the unit of substance but the unification of a system. The main fault by which a spiritualistic soul-conception is vitiated consists in the materialization of the soul as if it were a thing or a bodily entity which at death takes wings and is subject to the law of matter that limits it to a definite existence in space. We have seen that spiritual life is not tied to space, and though it is definite and of a typical character it may be in several places at once.

Professor Fechner's idea of the soul suggests the assumption of spiritualistic conceptions, and he goes so far as to look upon abnormal soul-life as an indication of that higher form of existence which awaits us in the life to come. Here is the place where we cannot agree with his propositions although considering his inclinations we must confess that he has made a moderate use of this line of argument.

* * *

We may be permitted to recapitulate our view of the nature of the soul as being the significance of sentient

symbols. The soul originates from sentient substance. Sense impressions are being made, and in repetition the same sense impressions are felt to be the same. Hence a feeling of sameness is produced and this feeling comes to represent or symbolize the cause that produces the sense impression. Thus the soul grows naturally wherever sentiency exists, and from the variety of feelings which have become representative of the various sense impressions of the surrounding world, becomes the soul of man by the formation of language. With the help of words things are classified. Abstraction and generalization become possible and reason throws a flood of light upon the laws of being. The essential feature of soul-life is not, however, its tendency to classify and comprehend things, but the practical application of reason, its ability to direct and predetermine the course of events and so to become master of the situation. In a word, purpose is the characteristic feature of soul-life.

In this interpretation of soul which brings forth the salient feature of its nature, we see at once that substance or material, though indispensable, is of slight importance, and our psychology need not enter into details of physiological processes. We see, too, that the efficiency of a soul consists in the transfer of symbols which are understood by other souls. Whether or not the essential part of cerebral activity be constituted by the interaction of nitrogen and oxygen, or whether the ether and electric processes play an important part, is quite immaterial and can throw no light on the nature of the soul. At the same time it disposes of the assumption that a soul may consist of some sublimated substance of imponderable matter, and that this substance remains preserved in afterlife as a kind of ethereal personality which according to ancient belief was ascribed to spiritual beings such as angels, ghosts, and gods.

But at this point I hear the anxious question: If the soul is not somehow substantial, if it is merely the purpose and character of the will and the ideas which we are thinking, is it not then a mere nonentity which leaves us nothing but a barren nihilism? My answer is: Not at all. It is the nature of the spiritual not to be material, and substance of any kind, be it conceived as ever so refined or sublimated, remains material; and any soul-conception which has not worked itself out into the clearness of conceiving the soul as pure form, falls short of the truth. But I grant that man is still so materialistic in his conceptions that he stands in constant need of materialistic allegories, and the soul has been represented in art since the days of Egyptian culture as a winged figure of luminous nature, visible only to the eye of the spiritually minded. All this is mere poetry which however develops naturally in man's imagination, and though the soul is not an astral body, not a luminous substance, not a sublimated entity of any kind. it is in itself real enough.

The essential feature of the soul is form, for form constitutes the type, and the form of the soul is character. Not that we feel is of importance but what we feel; not that we live, but how we live. But man wants life and soul and spirit to be substances as thick and clumsy as rocks, and as gross and coarse as our bodies; and there are many who imagine that if form is the essential feature of existence, our philosophy will end in nihilism.

Schiller castigates the sensualism which dominates the opinions of vast numbers of religious people, in a couplet called the "God-eaters," or *Theophagi*:

> "How they seek pleasure in all! These gluttons are eating ideas.
> Spoons they will carry, and forks, into the kingdom of God!"

Theological hedonism is the secret reason of all narrowness; it is sensual to the core and would not brook a

spiritual conception of the letter of religious dogmas. Thoughtless people cling to the allegory of the parable because it pleases the senses, and scorn its significance because the spirit escapes them. When sensualism no longer rules supreme, the time will come in which a philosophic conception of religion will be better appreciated.

Let us learn that form, though not a substance, is the reality of life, it is the superreal. The value of life lies not in its length, not in the number of years, not in the amount of happiness attained in it; but in the *direction* in which it tends and the *purpose* which it serves. Life in itself is a boon only as an opportunity to perform a task, to accomplish a certain work, to actualize an ideal. The aim of life is its significance, and that alone establishes its worth. By having an aim that is rooted in eternity, we need not mind the transiency of life. We can make it a revelation of imperishable ideals, and can impart to life a significance that is beyond the intrinsic meaning of the moment, and possesses a worth everlasting. To attain that worth everlasting is the aim of man's life. That is our immortality and in it we join

"The choir invisible
Of those immortal dead who live again
In minds made better by their presence."

* * *

Professor Fechner's favorite idea consists in the theory that this life to come of man is accompanied by an individual consciousness, and his assumption is based on the fundamental doctrime of psychophysics which proclaims the parallelism of psychic and physical processes.

The theory of psychophysical parallelism is practically accepted by modern science, though some objections have been raised to the word "parallelism" because, as has been said, we have not to do with two parallel series but with one process which allows two aspects, and the parallelism

is not one of reality but merely one of two abstractions. Be that as it may, we will grant that the theory if properly understood need not be regarded as dualistic and we do not hesitate to accept it in its main outlines.

According then to the theory of parallelism, feelings are the inner or psychical aspect of what in the outer or objective world appears as a cerebral commotion, and we observe that only those cerebral functions rise into consciousness which rise above a certain degree of intensity called the threshold (*limen*) of consciousness. But those nervous processes which are too weak to be felt are none the less real and effective in the whole system of our psychical activity. And we must assume that even inorganic nature is accompanied by some element of internality or subjectivity which in its climax corresponds to our consciousness. We have called this weakest group of mere subjectivity, potential feelings, and declared that all reality consists subjectively of such potential feelings which, wherever they assert themselves by action, are felt to be material bodies. Everything that exists is, subjectively, feeling or potential feeling; objectively, materiality or bodily existence. From within and to introspection we are soul; from without and to observation of others we are concrete bodies. (For further exposition see the author's *Soul of Man,* especially the first chapter, "Feeling and Motion.")

Now Professor Fechner claims that the parallelism cannot be limited to our bodily system but extends over the whole world—a consequence in which we follow him, and which, as just explained, is but the matter of obvious consistency. But he goes so far as to declare that the pure relations which obtain in other conditions are as much accompanied by consciousness as are the functions of our brain. To him the totality of the earth is an ensouled body and in addition to the consciousness of individual

beings that live on its surface, it is possessed of a soul of its own. The creatures that people the earth are comparable to nerve-cells of a human body and though every cell in our brain has a life of its own, the whole brain has a superadded consciousness which pursues its own plans and purposes. So, argues Professor Fechner, the earth is a living creature, a kind of super-personal being endowed with a peculiar consciousness in which we unconsciously participate. The same is true of other planets and of the universe as a whole, the spirit of the universe being God.

I will grant to Professor Fechner that the realities which constitute the character of the earth as a peculiar type of existence distinct from other planets are real and efficient but I would regard it as fantastical to look upon the sum total of those realities as ensouled beings—at least ensouled in the human sense, which is the commonly accepted meaning of the term. I deem it a pleasant allegory which will drive home to us the significance of these great interrelations of nature and the universe, and will serve as an evidence that they are by no means nonentities or negligible quantities. Goethe personifies the earth spirit who appears to Faust and characterizes himself in these lines:

"In the tides of life, in action's storm,
A fluctuant wave,
A shuttle free,
Birth and the grave,
An eternal sea,
A weaving, flowing
Life, all-glowing,
Thus at Time's humming loom 'tis my hand prepares
The garment of life which the Deity wears!"

But the personification is not to be taken seriously. Otherwise we would have to look upon the gods of Greece as real personages endowed with life and consciousness. We do not deny that wisdom is a reality though Athene,

the goddess of wisdom, is a mere personification, and though we would insist on the importance of the idea represented by the goddess. The same is applicable to all ideals of life whether or not they have been personified, and it is the main burden of our message to the world that these purely formal relations are the most significant realities of life.

Here then is our agreement with, and difference from Professor Fechner's theory in a nutshell. With him we believe in the reality, the significance, the all-importance of man's life after death, but we regard his description of the part which consciousness plays as misleading. We can not doubt, at least in many cases, that the personality of a man remains centered around his name, and continues to constitute a unit of its own. Yet the nature of this unit is not the same as the unity of a living person. It is purely spiritual and not physical—not physiological. Professor Fechner seems to regard man's being as endowed after death with the functions of the entire nervous system, including the sense-organs and the brain, for which assumption there is not the slightest warrant in ascertainable facts.

EDITOR.

A SCIENTIFIC SKETCH OF UNTRUTH.

THE chief object of this article is to show untruth in its general relations to scientific principles: its origin, necessity, utility, justification, and limitations; how it arises in the human brain, and how it affects human progress. In previous publications I have submitted views on kindred subjects, viz., *The Scientific Basis of Morality*, (Sonnenschein, 1899); "How the World is Governed," (*The Empire Review*, February, 1903); "The Coming Scientific Morality," (*The Westminster Review* and *The Monist*, April, 1904); "A Scientific View of Consciousness," (*The Monist*, April, 1905); "How Universal Energy Governs Mankind," etc. In these publications I have adduced evidence to show that the rudiments of our thoughts and actions can be traced down and merge into the properties and changes of the material elements of which we and our environments are composed. In consequence of the complexity and comprehensiveness of the subject the present article requires attentive reading.

I. GENERAL SCIENTIFIC RELATIONS OF UNTRUTH.

By the terms "truth" or "knowledge" I mean ideas which are consistent with all known facts and with all conclusions properly drawn from them; and by "untruths" I mean statements which are not consistent with facts. That is not knowledge which is not true. It is a strict rule in real science to limit belief to the evidence, and to

exclude all inconsistent ideas, otherwise truth could not be attained, but our mental faculties are so weak and our lives are so short that in ordinary affairs this rule is often disregarded. The human brain also is so constituted that it retains almost any impression, whether true or false, which is strongly made upon it, and we therefore often believe, though we do not really know that which is untrue. The mere antiquity of a belief is not a proof that it is either true or permanent, the ancient one that the sun moved around the earth is an example.

Next to life itself truth is our most valuable possession. As mere belief is often treated as truth, multitudes of persons are willing to suffer almost any pain for the sake of their beliefs, whether true or false, and under this influence many have sacrificed their lives as martyrs, missionaries, explorers, and discoverers. The chief explanation of this fact is that a strongly-fixed cerebral impression is part of the nervous structure, and like a mark of small-pox, is constantly renewed as it wears out. The strength of some fixed beliefs is so great that under their influence thousands of innocent witches and heretics have been slaughtered and burned alive by believers in unprovable dogmas. In consequence, also, of the incessant action of our environment upon us, our love of pleasant falsehoods is so strong that new ones are frequently invented to gratify us. The extent to which we accept and practice untruth indicates the degree of uncertainty of our ideas, and the limits of our love of truth.

As truth and untruth are ideas almost wholly confined to human beings, they are only manifested by an extremely small portion of nature; but they seem large and important to us because they affect us so personally. Science has proved that we are mere specks of matter in the cosmos, animated by its all-pervading motion; we come and go, but the world continues, and we only possess anything for

a lifetime, a mere speck of eternity. Each man perceives only a very minute portion of nature, and understands immeasurably less, and the more ignorant he is the more familiar he assumes to be with the greatest mysteries. The comparative unimportance of man in the universe is shown by the extent to which he is sacrificed by great natural actions, such as earthquakes, volcanic outbursts, hurricanes, diseases, etc.

The human brain is only partly developed, and our intellect is so weak that in nearly all complex questions such as that of morality, we are often compelled to guess conclusions, and this is especially the case in apportioning punishment or reward. The extreme feebleness of our senses and reasoning-powers, even with the aid of scientific appliances, limits nearly all our ideas. There is consequently a maximum of knowledge beyond which we cannot attain, and all additional unknown truth is to us an infinite void. A photographic surface is in this respect similar to the human brain, in that all beyond a small portion of the solar spectrum makes no image upon it, and is to it an entire blank. Further, as we estimate nearly all things by our own little selves, the pains we endure prevent our taking a true view of our position in nature, and the immense amount of human misery around us blinds us to the fact that untruth and pain are largely necessary to human progress and development. In addition to the very limited power of our intellect, and the inconceivable vastness and complexity of nature, nothing is either fixed, final, or quiescent; all things are moving, our mental condition is always changing, and it is largely in this way that many enquirers after the truth, especially in psychical questions, are baffled in their researches, they cannot realize all the possibilities of error. Varying degrees of probability also limit all our thoughts, and we cannot venture safely beyond them.

In addition to these causes of untruth, very few persons think alike except in well-verified science; there are also two or more different aspects of every moral, social, and political question. It requires broad views as well as narrow ones to get through life safely, the narrow view is—"this is a wicked world," whilst the broad one is—"whatever is must be, under all its conditions and circumstances." Nature is full of seeming contradictions, but has no real ones, and we are compelled by our environments to believe both truth and untruth. Our capacities of receiving truth and untruth depend largely upon the properties of our nervous system; thus, dullness of the senses considerably prevents cerebral impressions. In interpreting such impressions, we are usually actuated more by feeling than by reason because our environments act first upon the senses, and only through them upon our intellect, and power is lost in the process. Our feelings are our first guides, and act automatically. We imagine both truthful and untruthful ideas spontaneously to a very large extent; and there are many persons who live almost entirely automatically, without the help of much knowledge or intellect. It is not so much because an idea is true or false that we fail to accept it, but because the brain is unprepared to receive it, or because it contradicts our previous ideas and confuses us. The ability to receive truth depends largely upon the kind and amount of knowledge already possessed, thus he who does not know the alphabet cannot read; similarly, unscientific persons cannot readily accept the scientific basis of morality.

Notwithstanding that untruth is regarded as a species of "sin" to be "stamped out by means of religion," do what we may, we cannot prevent our environments producing it, nor alter the principles upon which the universe and mankind are governed, and which lie at the root of the matter. The only effective preventive of untruth is the

gradual discovery and diffusion of truth, and this can only take place at a very slow rate, because rapid improvement of human beings is more than they can bear.

An extensive examination of books and articles in reviews, etc., on moral and theological subjects, shows that the entire topic of "untruth," "sin," and "evil" remains even now a tangled mass of vague complex statements, which no man can clearly understand. Nearly all such publications are largely written as if the fundamental powers and principles of science have no existence and exert no influence upon man's moral ideas or actions. This is largely due to the difficulties of the subject, and to our feeble apprehension of fundamental science — a special professorship of the chief principles of science in each university might perhaps help to remedy this omission. To live without adequate knowledge of the powers which govern the universe and mankind is largely to "live without God in the world"; but we have little choice in the matter.

In some exceptional cases even clever scientific men seem to lose their reason when they attempt to harmonize science and dogma. Thus Faraday, when asked how he could believe theological doctrines inconsistent with science, replied, "I prostrate my intellect." It is, however, a conspicuous fact that nearly all of the thousands of able scientific discoverers have avoided such subjects, largely because of their confusing nature, and especially because of the great number of untruths and inconsistencies associated with them. The attempt to harmonize science and dogma is nearly always a failure, and whilst we may consider unproved statements as such, in order to test them, we cannot with safety fixedly believe important ones without proper and sufficient evidence, and if the evidenve is insufficient we must wait for more.

Inconsistency is a degree of untruth and often a first

sign of insanity; no man can believe in dogmas, miracles, ministering angels, the existence of spirits in space, life or mind without matter, etc., and at the same time in the fundamental principles of science, without suffering cerebral confusion, because the two sets of ideas are contradictory. A proof that the methods employed by scientific discoverers, viz., experimental observation, meditation, and inference, are reliable, and those used by supernaturalists are not, is the fact, that whilst tens of thousands of new truths have been found by means of the former, few or none have been evolved by the help of the latter. The great failure of untruth as a means of discovery does not, however, prove that it is useless or entirely unjustifiable.

2. THE ORIGIN OF UNTRUTH.

Of what is termed the "first cause" of truth or untruth we know nothing, simply because it is beyond the limits of human power:

> "Heaven from all creatures hides the book of fate,
> All but the page prescribed, their present state."
> —Pope.

Any natural action, such as thought, is only one in an infinite series of events, and our perceptive power is altogether too limited to enable us to trace our ideas back to a "first cause" in the infinite past. The deepest cause, both of truth and untruth, so far as we can at present trace it, is the universal motion within and around us, which is always acting upon our cerebrum and producing impressions. Physicists well know, from the phenomena of astronomy, spectrum-analysis, heat etc., that all substances, whether living or dead, are in this state of perpetual and indestructible motion; and that this motion acts upon all things by means of radiations through the ether. It acts upon men, and compels each man to influence his fellowman, and his fellowman to influence him.

The origin of untruth depends further upon the fact

that all bodies, whether living or dead, have but limited powers, and that the intellect of man can only imperfectly distinguish truth from error; this human feebleness is largely due to the small range of the senses, the immensity of the universe, and the almost infinite complexity of nearly everything in it. The smallest irregularity, or weakness of action of the senses or intellect, also produces false ideas, and it is largely in this way that untruth and "sin" enter the human brain. A multitude of untrue impressions arise from the circumstance that we magnify the importance of everything we feel, and pass unnoticed the greatest of truths if they do not affect our consciousness. Even the life-supporting substance, oxygen gas, was not discovered until the year 1774.

It is well known that truth is mingled with untruth in every person, but it is not so obvious that the essential source of each, viz., the impresions produced upon the brain by the motion of its environments, is the same in all. The result produced is either truth or untruth according to the kind of cerebral impresions made and of those already present. If the incoming ideas are true and the brain is properly prepared, truth is received; but if either the new ideas are false, or the brain is unprepared, untruth is produced. Many impressions are made upon our brains without our perceiving them, and are occasionally revived in our dreams. They are often untrue because our intellect is not sufficiently awake to correct them. In consequence of differences of perceptive ability and of existing impressions in different persons, that which appears true to one person often seems untrue to another, thus the dogma of "The Immaculate Conception" appears true to Roman Catholics but is utterly false to physiologists and scientific men generally.

The reception or rejection, both of truth and of untruth is largely dependent upon the intellectual state of the brain.

Thus boys and girls are not admitted into higher-grade schools if they have not been properly prepared in lower ones; similarly, the brain of each person must already contain a sufficient number of true ideas suitable to correct the incoming ones by means of comparison and inference. It is largely through deficiency of truthful impressions and of suitable training, that ignorant persons are frequently untruthful. The universe is infinitely truthful, and the chief sources of untruth are in us. The greater the number of important truths we know, the smaller, usually, is the proportion of untruths we believe, because each important truth corrects and weakens some untruths. We are immersed in an unlimited ocean of mysteries, and in consequence of our very limited faculties, we believe a multitude of untruths respecting them.

The reception and rejection of truth and untruth are also largely dependent upon sensation and feeling. As our environments act first upon our feelings, and only secondly upon our intellect, the way we receive our impressions is usually decided by the feelings; if these are pleasant they are accepted, and if painful they are rejected. The ordinary automatic excitement of our brains, uncorrected by evidence and intellect, is a great source of untruth; thus we spontaneously imagine all kinds of false ideas. We fancy that we are stronger than nature, because of our numerous discoveries and inventions, forgetting that it is the energy of our surroundings which causes us to make these discoveries, and that nature has vastly more power over us than we have over it. We imagine that a "spirit" or an "ego" within us draws our conclusions, whilst the fact is, our conclusions are produced by the influence of the evidence and of our feelings upon our intellect, the cerebral impressions automatically causing the inferences by the aid of comparison. We fancy that our volitions are due to the "ego," not thinking that they may be produced

by unobserved stored-up cerebral motion. It is only by automatic action and re-action, accompanied by transference of energy, that we to a small extent "govern nature" by our volitions;—similarly, in our ideas of "choice, selection, and self-guidance," we overlook the fact that we are automatically compelled to perform them. It is well known that metals automatically "select" particular metals from mixed metallic solutions, and that crystals automatically "choose" pure ingredients from impure saline liquids with which to build up their structures, and therefore a mysterious "power" is not necessary to perform the same kind of work in us. Energy is always stored up in us from our food, and liberated under suitable conditions, and the so-called "soul in atoms" and "mind in plants" are probably different modes of motion in those different bodies. Neither the sun, nor universal energy is a "divinity," simply because it guides and governs terrestrial and heavenly bodies.

Some persons believe that "ministering angels" and beings of a higher order than man surround us, that vitality exists independent of matter, that space is tenanted both by human ghosts and vital ones, by spirits of dead plants and animals. But we do not possess satisfactory evidence which proves that these ideas are true; and such beliefs would be prevented by adequate knowledge of physiology. The radiation images emitted by substances, and which travel 186,400 miles a second, cannot be permanent spirits, because they are transformed, and cease to exist, when they fall upon other bodies. Another common error is that human thoughts and actions are "guided" by some "spiritual power" outside them, but the discoveries made by Newton, Joule and others, of the planetary movements especially, have largely proved that the universe is a self-guiding machine; that the energy in each act of change passes automatically from one substance into another and is altered in its form in the process, but without perceptible

loss or gain in total amount. Therefore there is no work left for "spiritual power" to perform. Such spiritualistic hypotheses are too complex and vast to be proved by mere human observation alone, they require a variety of well-verified evidence. Nothing less than the incorrupted testimony of inanimate substances, such as are employed in physics and chemistry, would be sufficient to fully justify us in firmly believing such important and extraordinary statements; and we must wait patiently for such evidence respecting them as Newton waited for Picard's measurements to prove the truth of the law of gravitation. The number and variety of forms in which untruth automatically arises in the human brain is practically endless, but the instances given are sufficient.

Suitable knowledge of science enables us to detect the rudimentary causes of nearly all our chief actions in plants, and even in minerals. Magnets and metals show distinctly the rudiments of heredity and of memory, and crystals show those of "choice" and "volition." Man is not the only deceiver, many other animals deceive, and although inanimate bodies do not actually utter untruths, their properties and actions suggest them. Thus the sun, during all ancient time appeared to revolve round the earth, until astronomers discovered the deception; and the weather is notoriously deceitful. The rudiments of various "moral and immoral" qualities in plants have been described by J. E. Taylor in his book *The Sagacity and Morality of Plants*. The statements made respecting the existence of "souls in atoms" and "mind in plants" are largely instances of incorrect nomenclature. Calling things by false names is common practice, and energy is not "mind" simply because it is motion.

Untruth is often an immediate result, either of ignorance or design. Many serious untruths have been invented and promulgated at different periods. Those of

the existence of a "heaven," a "hell," a "devil," and "immortality of the soul" are very ancient; that of "transubstantiation" was invented in the ninth century, that of the "Immaculate Conception" was made an article of faith in 1854; and the "Infallibility of the Pope in faith and morals" was decreed in 1870; and some persons even now venture to assert that "the value of a single soul is infinite." Untruth is greatly promoted by human conceit; in consequence of the incessant action of our environments upon our senses and feelings, the importance of everything human and personal is exaggerated. Thus we consider that "the universe was made for man," whilst it is incomparably more true that man is merely a denizen and an extremely minute fraction of this little globe. It is altogether beyond human power to adequately perceive its own littleness in the cosmos. The universe is one of truth, and the cause of our inability to truly explain it lies largely in us; we are simply deceived because we are ignorant. It is chiefly in consequence of the inconsistence between ordinary narrow views and comprehensive scientific ones that the entire subject of "sin and evil" is a vast paradox to unscientific persons.

3. THE JUSTIFICATION OF UNTRUTH.

Absolute compulsion justifies all things, even the immense destruction of human beings by great terrestrial catastrophes, epidemics, etc., and no exceptions are made in favor of men, whatever their sufferings or beliefs may be. The supreme question throughout nature is, not what is most truthful or pleasing to mankind, but what is most necessary to the well-being of the universe, and everything yields to this. The first effect of universal natural energy is to preserve the cosmos, and this is the greatest good because all other good things depend upon it; if this world was destroyed all mankind would perish with it.

"The first Almighty Cause
Acts less by partial than by general laws."
—Pope.

. . . . . .

"Better for us, perhaps, it might appear,
Were there all harmony, all virtue here;
That never air or ocean felt the wind,
That never passion discomposed the mind,
But all subsists by elemental strife
And passions are the elements of life."
—Pope.

With every living thing there is action and re-action; life is a struggle and a conflict. The fox gains a living by means of deceit, and the incomes of many men and women are largely secured by means of untruth and unceasing competition.

The strongest power governs in nearly all cases; the stomach must be fed before the intellect, because it is the stronger influence. Our most pressing question usually is, not what is true, but how can we live and enjoy life. We are primarily governed by infinite energy, and our thoughts, whether true or untrue, are as necessary consequences of natural powers as the actions of inanimate substances. We possess no full proof that an act of thought is in any case an exception to this, but we have abundant evidence to show that our ideas require expenditure of natural energy to produce them. The formation of each truth or untruth in the brain is attended by consumption of heat. Each single idea also arises out of its immediately preceding cause, in accordance with the principles of continuity and equivalency of energy. We all think and act as we are compelled by our environments, and we cannot much prevent them forcing us to believe untruth as well as truth.

An opinion has long been held, though rarely expressed, that there exists some degree of justification of untruth, and this unpleasant conclusion is largely supported by evidence. Thus the simple facts that untruth

is a necessary consequence of the action of our environments upon us, and that each man thinks as he is compelled under all his conditions and circumstances, are alone sufficient to prove some degree of justification for it. The opinion is further supported by the fact that truth and untruth are essentially alike. Thus both are cerebral impressions caused by natural influences; each is limited by circumstances; each supplies pleasures and pains, consolations and disappointments; each is necessary to human progress; the use or abuse of either is rewarded or punished; and we justify each so far as we think it does good. There are all degrees of truth and untruth. Each has opposite effects according to its amount, so that if either is deficient, it does not sufficiently stimulate us to action, and if in excess it excites us too much. Both are governed by the influence of our environments, and are kept within certain bounds by their conditions. A due amount of truth promotes human progress, a deficiency fails to stimulate it, and an excess retards it by producing too rapid a change. Untruth aids progress by exciting conflict and discussion and retards it by misleading mankind.

There are, however, conspicuous differences between them, and although these are of very small value in the cosmos, their highly personal effects cause us to feel them keenly, and to exaggerate their degrees of importance. The chief ones are that truth is reliable, whilst untruth is not; truth is a safe basis for thought, prediction, and action, but untruth is always liable to be detected, refuted, and rejected; we cannot reliably predict future events from untruthful data; truth leads to new knowledge and scientific discoveries, but untruth does not; truth is consistent with all known facts, but untruth is not; untruths are often cheap at the outset, but dear at the end; while truths are the reverse. Truth is far superior to untruth as a guide of life, and as a consolation in times of distress, be-

cause it is trustworthy and unalterable. It is, however, much more difficult to obtain. The reliability of a man's word is largely proportional to his freedom from untruthful beliefs, because they prevent the reception of truth; and to his knowledge of the great principles of science, because they are the chief tests of truth and error. True views of nature and of human conduct yield the most permanent happiness. It is largely in consequence of deficiency of such knowledge that untruths are believed and propagated by many of the kindest men. Truth is the best stimulant to self-improvement. The acceptance of it is largely a question of education and training, and of how much self-sacrifice and labor a person is able to bear. Whilst truth imparts courage and self-reliance, the uncertainty of untruth weakens all our powers. Whilst too much truth is more than ignorant persons can comprehend, too much untruth is more than intelligent ones can bear, whence arises continual conflict between these two sections of mankind.

Untruths are fundamentally mere temporary expedients in the evolution of man. They are often unhealthy and a first step to insanity, and as untruthfulness often leads ultimately to suffering, we should be as careful about what ideas we accept as we are of what food we partake, and adopt similar general rules in accepting them. In consequence of untruth and ignorance, millions of persons are annually led into disease, misery, crime, insanity and early death; and it is only on acquiring sufficient knowledge of the chief powers and principles which govern the universe, and refering all questions to them for explanation, that untruth can be prevented from entering the human brain. Untruth is not accepted if we are trained well and possess, in advance, the knowledge necessary to correct it. But to entirely prevent it is impossible because that would require unlimited knowledge; the utmost we can do is to

correct and slowly diminish it. As all our attempts at avoiding it are extremely feeble in comparison with the magnitude of the task, we go on much in the same way from one generation to another. All mankind cannot be suddenly made truthful, and even if they could, each generation would still require teaching afresh. Belief in untruths generally must continue as long as mankind remains ignorant of the chief principles of science. Untruth, conflict and strife are inseparable from human life. Conflict is a result of motion; the whole of nature exhibits it, thus waves beat upon rocks and wash them away. All nature appears deceitful, but the real source of deception is usually the feebleness of our faculties.

Notwithstanding its drawbacks, untruth is a cosmic and human necessity. It is necessary as an element in the general evolution of man. We are in a state of gradual development, and require some stimulus to compel us to change. Truths and untruths, pains and pleasures stimulate us; they strengthen us if accepted rightly and are not too great for us to bear, and weaken us if in excess. The facts also that untruthful expressions are produced by the same natural process as truthful ones, and exist in accordance with the law of universal causation, are alone sufficient to prove their necessity. There exists some degree of virtue even in untruth, but our views are so narrow we can hardly believe it, thus, in common with other kinds of "evil" it is useful as a regulator of human progress. As deadly poisons, such as arsenic, prussic acid, and strychnine, are sometimes necessary to relieve disease of the human body, untruth may in some cases be necessary and justifiable to relieve distress of the mind. Through lack of knowledge and courage we frequently consider pain and untruth to be "evil" when they are not. The idea that truth and untruth are really evil when they produce pain, will not bear examination; the universe does not exist

merely for our pleasure, but we exist to obey omnipotent power. A large portion of the pain of life arises from our ignorance and false idcas, and another from great natural causes over which we have but little or no control, and all animals have to submit to it.

According to the widest scientific knowledge "whatever is, is right," but according to ordinary narrow ideas "this is a wicked world," and "there is something wrong in everything"; but even this dissatisfaction is useful because it excites discussion and stimulates change and progress. Untruth is only one form of a multitude of actions we term "evil," but according to science all these actions, ranging from the greatest physical catastrophes to mere exaggeration in men, are ultimate results of physical powers, and men are as perfect as the fundamental properties of the bodies of which they are composed allow them to be. Man is not imperfect because he is complex; a complex ocean-steamer is as perfect as a simple sailing-ship. Nor is he "sinful" because he is weak; his "sins" and the actions within and around him, arise out of the facts that all substances and creatures act, or fail to act, upon each other, according to their natural properties.

> "Then say not man's imperfect, Heaven's at fault,
> Say rather, man's as perfect as he ought."
>
> —Pope.

and that when all is known and all is understood, mankind will see "whatever is, is good," and that man is working out his destiny by means of untruth and of truth.

Untruth has often more adherents than truth in complex subjects, chiefly because false explanations are frequently more pleasing or easily understood than true ones; and the question what is justifiable in any complex case, is never finally settled to the satisfaction of all persons, simply because very few persons think alike in complicated subjects. Nearly all persons practically support by their

conduct the conclusion that untruth is more or less justifiable. The amount of pleasure and stimulus afforded by it in tales, novels, plays, poetry, suggestive music, oratory, sermons, allegories, metaphors, dogmatic beliefs, miracles, pseudo-science, etc., is immense—the popularity of Bunyan's Pilgrim's Progress and Robinson Crusoe are familiar examples. And the consolation afforded by theological dogmas to millions of persons in distress or near to death, is abundant proof of its immediate utility, whether it is fully justifiable or not. The chief explanation of these facts is, that we are largely compelled to tell untruths by the influence of circumstances upon us, and because it is often a habit or personal advantage. It is of great use in helping us to acquire money, social influence, popularity, and success. A vast amount of it is employed in nearly all occupations, in trade, manufactures, literature, social appearances, sectarian teaching, etc.: it has been used to an immense extent in all ages to satisfy the almost universal desire for pleasure, even the prophet Jeremiah is reported to have said, "the prophets prophesy falsely, the priests rule by their means, and the people love it to be so"; and as untruth is so useful, it necessarily continues. Most persons tell untruths spontaneously, as far as they think it is for their advantage, as far as they dare, and as other persons allow them; and all are more or less occupied in defending themselves from the untruth and deceit of their fellows.

The same universal energy which produces changes throughout nature, compels us to use both truth and untruth in securing a maintenance. As life is and must be our first object, and untruth is so necessary in obtaining an income, we may not entirely condemn those who practise or diffuse it, but there is a limit to its justification, and whilst we should be tolerant towards others, we must protect ourselves from injury, both by untruthful persons, and

by advocates of too rapid advance; we require to be duly altruistic towards all men, and sufficiently conservative towards ourselves. Absolute necessity largely justifies the conduct of a criminal as well as that of the judge who condemns him to punishment. Persons are blamed and punished, not so much for their misdeeds as to deter them from doing wrong in the future. The fact that untruth is more or less necessary, justifiable, and good for mankind is one of the many painful truths we have to accept, whether we are willing or not. Similar arguments justify in various degrees all that we term "sin and evil."

The real scientific justification of an act is the degree to which it contributes to the welfare of the cosmos, therefore to that of all men, and not alone to our personal pleasure. The well-being of the universe, and "the greatest good" are nearly synonymous, and the noblest object of life is comprehensive utility. There are all degrees of "goodness," and that which does good, is more or less good whether it be a truth or an untruth, and the larger the "good" and the less the "evil" produced, the greater is the degree of justification. But "good in excess is evil," and extreme pleasure is painful. An excess of truth or of pleasure is more than some persons can bear, and much of the cheap goodness extant is hardly worthy of the name. There exist nearly all degrees of justification of untruth, ranging from that of automatic unintenional exaggeration to that of serious manifest falsehood. Such automatic falsehood is very prevalent, is often acquired by habit, and is a frequent sign of ignorance and defective training. But if some persons did not readily exaggerate the importance of money and pleasure they would not be sufficiently stimulated to obtain it, and if others did not over-value new knowledge they would not make the sacrifices necessary to discover it, and human progress would be impeded. As also the brains and nerves of different persons require

different degrees of stimulation to excite them, exaggeration is in some degree useful. In consequence of our very limited intellect in complex cases, some persons practice expediency and try to "appear all things to all men," but even this is a degree of untruth, and has some justification. It is only by faith in the greatest truths that we can be fully justified, and the most final court of appeal is consistency with all known facts.

How far untruth is justifiable in any particular case depends upon the circumstances, and as we cannot actually measure the value of these, we are only able to guess a reply, and act upon the degree of probability, and every different person guesses differently. This difficulty is usually diminished by employing juries, committees, etc., on the rule that "in a multitude of counsellors there is safety." Such cases are almost endless in number and variety, and give rise to continual discussion without arriving at any final conclusion. Justification of untruth is largely a personal matter which each man must decide for himself and accept the consequences. The practice of untruth, even for laudable purposes, lessens respect, because it shows our feebleness; and how far it is justifiable to believe and teach unprovable statements in important subjects is a serious question, and requires frequent revision as civilization advances.

4. LIMITATION OF UNTRUTH.

The general restrictions of untruth are much the same as those of truth: viz., how much our environments compel us to accept, and how much we are able to bear. As the properties and powers of nearly everything are limited, this restricts the effect of our surroundings upon us. Too much truth stuns us, and too much untruth repels us, and each excites our resistance. The untruthful man does as he must, and his victims do as they must in return in order

to protect themselves and restrain his conduct. The fact that both truth and untruth are kept within bounds by our feelings, shows that they are governed and limited by the stronger influence of our environments. Everything limits the action of everything of opposite property. Truth and untruth limit each other; intelligence and ignorance, ditto; the same with science and dogma; and with the advancing and retarding sections of mankind; we cannot simultaneously think of two opposite ideas; and even a Deity could not be infinitely just and infinitely merciful. Both truth and untruth are limited by rules of social conduct. Ordinary social life could not be carried on if untruth was too prevalent, or if too much new truth was suddenly made known. If ignorant persons had too great power there would be no supply of new knowledge and no human progress; and if the most scientific had too much control, the labor of self-improvement cast upon ignorant persons would be more than they could bear, even now millions of persons fail prematurely in the struggle of life. All these phenomena depend largely upon the great fact that every action requires time, and is almost wholly determined for us by natural powers.

Untruth in any subject usually lasts only as long as mankind is pleased or benefited by it. Old habits and false ideas die out; the dogma of all things being composed of "earth, air, fire, and water" has disappeared, and others are following it; even the view that the elementary substances are wholly inconvertible is being modified, for we now know that some of them change into others with extreme slowness. The use of truth is restricted in various ways similar to that of untruth; thus it may not be told at all times to all persons or under all circumstances; and if it is said with malicious intent, and not for public advantage, it becomes libel; untruth in cases of perjury is criminal. We must not use either truth or untruth so as

to inflict avoidable pain upon others, except in self-protection or defence: thus, although we may not speak the truth at all times or in all places, we may give evidence against criminals and confine them in jail, in order to ensure public safety and criminal reform. It is even good in some cases to keep other persons ignorant: thus doctors may not always tell the truth to their patients, lest it may delay their recovery; but they may mention it to their colleagues in consultation. Extremists in truth or untruth nearly always suffer; thus Bruno, Galileo, and nearly all advanced scientific men have been punished, whilst very ignorant persons are nearly always in distress. In all these cases universal energy acts so as to secure the greatest "good" to the cosmos, and compels each man to be more or less "a law unto himself," but "goodness" is largely unmeasurable.

The limitations and restraints put upon the use of truth and of untruth constitute a large portion of the subject of morality, and the natural explanation of each is largely the resistance of those persons who are pained by them. All our ideas are more or less limited by the sect, community, or nation, amongst whom we live—and we may not mention too freely truths which they dislike. We are surrounded by multitudes of circumstances which compel us to be either more or less truthful or untruthful. The number of untruthful ideas is greater than those of truth, and there are usually more ways of going wrong than right.

5. RELATIONS OF TRUTH AND UNTRUTH TO HUMAN PROGRESS.

The origin of human progress lies deep in the differences of internal motion of unlike material bodies. Each different person, or substance, by its different behavior or property, acts differently upon us, and produces our

growth, maintenance and decay. Food and friends support us, whilst poison and enemies kill us. The different influences act through our feelings of pleasure and pain; thus, through that of pleasure we are stimulated to desire more progress, and through that of pain we are compelled to avoid or diminish it. And in order to accomplish these objects we are driven to seek new knowledge; in this way, whilst our present information maintains our present state, scientific research evolves new knowledge for advance.

We only advance as fast as we are impelled. The rate of general progress is probably as certain and definite as that of the earth in its orbit, but owing to the immense complexity of the phenomena, and the limited extent of our powers, we are unable to measure it. The rate depends upon the fact that every action requires time; and upon the ability of the human body and brain to bear change. It is usually increased by truth, and decreased by untruth. It is largely regulated by the incessant conflict of the intelligent and the ignorant; whilst the former originate new knowledge and invent new applications, the latter prevent too rapid change. In accordance with the principle of equality of action and re-action, conflict is a part of our existence, and believers in truth and untruth constantly restrain each other. Untruth has two opposite effects upon human progress according to its strength; thus, if its strength is deficient, it fails to retard it, if in due amount it produces conflict and competition, and if in excess it retards progress by exciting too much resistance; an excess of new truths upsets our arangements, and it is well known that although we cannot measure goodness we may have "too much of a good thing."

The greatest exception to the use of untruth is original scientific research; this occupation stands alone as the search for truth by reliable methods. Whilst untruth is entirely forbidden in it, unlimited faith in unproved state-

ments is insisted upon in theological teaching. In all real scientific research, dogmas and hypotheses are treated merely as questions to be tested. Its reliability is practically shown by the success of its predictions, and by the variety of ways in which the governments of civilized nations depend for advice upon scientific societies; and its extent is partly shown by the titles of hundreds of thousands of costly researches contained in The Royal Society Catalogue, eleven volumes of which, costing more than £10,000 have already been published.

6. CONCLUDING REMARKS.

The principles set forth in this sketch constitute a scientific basis of morality, and offer great scope for application in explaining the evolution and government of mankind; they enable us to distinguish truth from error in a multitude of complex cases, and are capable of rationally explaining "sin and evil"; they form the truest philosophy and supply the safest consolation in periods of distress to those who can comprehend the subject, because they are consistent with all known facts. Their practical applications are very interesting, because they affect all men, and every act of human conduct. A fixed belief that all persons are governed by universal natural causation, and "do as they must under all the conditions and circumstances," requires us to forgive our enemies; and a full conviction that the most moral rule of conduct is "to do the greatest good," compels us to remember their actions, because knowledge is necessary for our guidance and defence. In this way, whilst knowledge of the great principles of science increases toleration of "sin," it also increases obedience to law, and excites a due amount of conflict and progress. Even the valuable Confucian injunction, to "do unto others as we would have them do unto

us" is not as correct as science requires, because it does not take into consideration the fact that what we would have done unto us may not be "the greatest good" for others.

The whole of the evidence supports the common opinion that it is largely beyond our power to devise a better system of nature than that which already exists, and which causes our evolution as well as our existence.

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CRITICISMS AND DISCUSSIONS.

HAECKEL'S THESES FOR A MONISTIC ALLIANCE.*

Professor Haeckel has written a series of theses, which are intended as a confession of faith for all those who would embrace a scientific world-conception, and take a firm stand on monism as contrasted with the traditional dualism of dogmatic Christianity and kindred religions of the past. He thinks that the time has come for all monists to form a great monistic alliance and these articles are intended to serve as the standard around which they should rally. Unfortunately the theses are so contentious that they must be irritating to the large masses who, though they are willing to follow science wherever it may lead, have not yet renounced their allegiance to the old religious ideals; and though willing to discard any errors of dogma or tradition, do not propose to surrender whatever good there is in religion.

It is further unfortunate that the formulation of the outlines of the scientific world-conception is rather slovenly and made without due precaution. For instance, the term *a priori* is introduced and expressly referred to Kant. Apriorism, as well as all assertions of metaphysics, is rejected and Kant's name is put in parentheses to leave no doubt as to whose philosophy is meant. This is done in spite of the fact that Professor Haeckel is very familiar with Kant's philosophy, to which he devotes a special chapter in one of his larger works. He should have borne in mind that Kant's doctrine of the *a priori* is quite in keeping with a scientific philosophy in the most rigorous sense of the word, and that Kant's metaphys-

* Prof. Ernst Haeckel's confession of faith, entitled *Universal Monistic Alliance, Theses for the Organization of Monism,* was presented before the International Congress of Freethinkers at St. Louis, Mo. A revised edition of the translation then under discussion has been published by The Humanitarian Review Office, 852 East Lee St., Los Angeles, Cal.

ics should not be confounded with the metaphysical systems of many of his interpreters and followers.[1]

In a similar way I take exception to his doctrine of determinism which excludes the possibility of free will.[2] I object to his treatment of the origin of man as being descended from the ape.[3] Presumably we agree as to facts, but we differ as to the propriety of the mode of presenting them. I am further not satisfied in having the oneness of nature limited to biology and physics, as if the realm built up by man (who with all his wealth of moral ideals and scientific attainments is simply characterized as a social *animal*) were a mere and insignificant variation of biological phenomena.[4]

But in addition to the carelessness with which these thirty theses have been drawn up, Professor Haeckel commits another mistake almost more grievous. He speaks of divine revelation as being based either upon lack of clearness or intentional fraud, a theory much in vogue among the rationalists of the eighteenth century but long since discarded by historians and investigators. He has not as yet learned to apply the theory of evolution to the domain of religion and the history of science. He appears to be unmindful of the fact that ideas pass through definite phases of growth, and that some errors are inevitable because natural, and that these errors contain a truth which has to be sifted from preliminary formulations by a slow process of maturation.

The mythology and dogmatism of the earlier stages of our religious development are not so much errors—let alone frauds—as transitory phases, in the same way that flowers and the unripe state of fruit are essential steps in the preparation of the seed.

Professor Haeckel ought to be the last man to disregard the evolutionary phase in both religion and science; and knowing well that evolution is the dominant law of life, he ought to have shown

[1] Kant's inventory of the *a priori* is practically the domain of pure reason.

[2] According to my definition, will is free whenever it can act according to its own nature, and not (as others have it) when it can will as it pleases. The contrast is between free will and compulsion, not between free will and determinism. Every act is determined, but acts of free will are determined by the character of the acting person, while acts of compulsion are not. To identify freedom of will with indeterminism is simply a confusion of thought, for there is no sense in the proposition that will, in order to be free, ought to be undetermined—an arbitrary product of chance. There would be no ethics without freedom of will. For further details see *Fundamental Problems*, p. 191 ff.; *The Monist*, Vol. III, pp. 86-87; *The Ethical Problem*, pp. 45-50.

[3] See the author's article "The Ascent of Man," *The Open Court*, March, 1904.

[4] I have discussed Professor Haeckel's position in the second part of a recent article "Theology as a Science," *The Monist*, Vol. XIII, p. 24 ff.

more consideration of and love for the past, which is preparatory of the present and the future. If he had done so he would have seen in the churches more that is good and would have endeavored to advance the essential and purify it of its unessential and transitory elements.

Having stated our general criticism of Haeckel's theses, there is scarcely any need of entering into further details and pointing out how mistaken he is in emphasizing the one-sided view of the naturalist against the traditional conservatism of the churches. Thus he appears as an enemy to the existing churches instead of rejoicing at their higher development into a new and truer interpretation of their dogmas, and, though perhaps without meaning it, he introduces into the scientific atmosphere of those of his followers who accept the results of science rather than its methods, a spirit of iconoclasm which if it became predominant among liberals would be lamentable in the highest degree.

A confession of faith ought to be positive and not predominantly negative. It ought to state what is, or is to be, believed and not to reject the belief of others, unless in very general terms and without entering into details. Professor Haeckel might as well have added to his denunciation of Christianity, a condemnation of the faith of fetish worshipers in the darkest part of Africa, or the superstitions that still prevail in many temples of the Celestial Empire, or especially all modern aberrations of New Thought, Christian Science and Freethought, for there are quite a number of wild theories which parade under this name and associate unhesitatingly with Haeckel's friends the Freethinkers.

If the time has come for monists to rally under the flag of a common confession of faith, let it be simple and to the point—a statement of a few sentences which casts no slur on either dualism or pluralism, and leaves the consequences to be drawn by others from its own principles.

The formulation of a confession of faith, be it for monists, freethinkers, rationalists, or any kindred spirits of progress, depends very much upon the occasion and purpose for which it is to be made; but whatever circumstances might lead to it, it seems to me always preferable that it be limited to a statement of principles and that it exclude special results of investigation as much as possible.

There is one point on which monists of every stripe and color can unite, and this is the recognition of science as the appropriate

method of determining truth. The highest authority in matters religious ought to be the truth, and we deem it our duty to search for the truth with the most rigorous method that science places at our disposal. P. C.

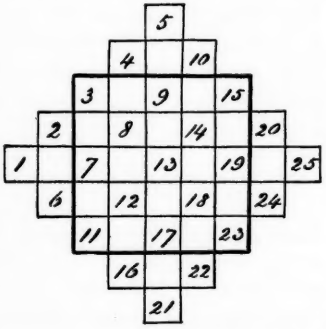
REFLECTIONS ON MAGIC SQUARES.

MATHEMATICAL, HISTORICAL, AND PHILOSOPHICAL.

Mathematics, especially in the field where it touches philosophy, has always been my foible, and so Mr. W. S. Andrews' article on "Magic Squares" tempted me to seek a graphic key to the interrelation among their figures which should reveal at a glance the mystery of their construction.

THE ORDER OF FIGURES.

In magic squares of odd numbers, 3×3, 5×5, 7×7, etc., there is no difficulty whatever, as Mr. Andrews' diagrams show at a glance. The consecutive figures run up slantingly in the form of a staircase, so as to let the next higher figure pass over into the



3	16	9	22	15
20	8	21	14	2
7	25	13	1	19
24	12	5	18	6
11	4	17	10	23

A SPECIMEN OF 5×5 MAGIC SQUARE.

next higher or lower cell of the next row, and those figures that according to this method would fall outside of the square, revert into it as if the magic square were for the time (at the moment of crossing its boundary) connected with its opposite side into the shape of a cylinder. This cannot be done at once with both its two opposite vertical and its two opposite horizontal sides, but the pro-

cess is easily represented in the plane by having the magic square extended on all its sides, and on passing its limits on one side we must treat the extension as if we had entered into the magic square on the side opposite to where we left it. If we now transfer the figures to their respective places of the inside square, they are shoved over in a way which by a regular transposition will counteract their regular increase of counting and so equalize the sums of entire rows.

The case is somewhat more complicated with even magic squares, and a suggestion which I propose to offer here, pertains to their formation. Mr. Andrews begins their discussion by stating that "in perfect magic squares of this class it is necessary that the sum of each column shall be the same amount, and also that the sum of any two numbers that are geometrically equidistant from the center of the square shall equal the sum of the first and last numbers of the series."

The smallest magic square of even numbers is, of course, 4×4; and he points out that if we write the figures in their regular order in a 4×4 square, those standing on the diagonal lines can remain in their places, while the rest are to be reversed so as to replace every figure by its complementary to 17 (i. e., 2 by 15, 3 by 14, 5 by 12, 9 by 8) the number 17 being the sum of the highest and lowest numbers of the magic square (i. e., $n^2 + 1$). It is by this reversal of figures that the inequalities of the natural order are equalized again, so as to make the sum of each row equal to 34, which is one fourth of the sum total of all figures, the general formula being:

$$\frac{1+2+3+4+\ldots.n}{n} = \frac{n}{2}(n^2+1).$$

We will now try to find out more about the relation which the magic square arrangement bears to the normal sequence of figures.

For each corner there are two ways, one horizontal and one vertical, in which figures can be written in the normal sequence; accordingly there are altogether eight possible arrangements, from which we select one as fundamental, and regard all others as mere variations, produced by inverting and reversing the order.

As the fundamental arrangement we choose the ordinary way of writing from the left to the right, proceeding in parallel lines downward. We call this "the ordinary order" or *o*. Its reverse proceeds from the lower right hand corner toward the left, and line by line upward, thus beginning the series where the ordinary

arrangement ends, and ending where it started. We call this order "the reversed ordinary," or simply *ro*.

Another order is produced by following the Hebrew and Arabic mode of writing: we begin in the upper right-hand corner, proceeding to the left, and then continue in the same way line by line downward. This, the inverse direction to the ordinary way, we call briefly *i*.

The reverse order of *i*, starting in the lower left corner, proceeding to the right, and then line by line upward we call *ri*. Further on we shall have occasion to represent these four orders by the following symbols: *o* by ●; *ro* by ⊛; *i* by ✢; *ri* by +.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

ORDER *o* (●).

6	5	4	3	2	1
12	11	10	9	8	7
18	17	16	15	14	13
24	23	22	21	20	19
30	29	28	27	26	25
36	35	34	33	32	31

ORDER *i* (✢)

31	32	33	34	35	36
25	26	27	28	29	30
19	20	21	22	23	24
13	14	15	16	17	18
7	8	9	10	11	12
1	2	3	4	5	6

ORDER *ri* (+).

36	35	34	33	32	31
30	29	28	27	26	25
24	23	22	21	20	19
18	17	16	15	14	13
12	11	10	9	8	7
6	5	4	3	2	1

ORDER *ro* (⊛).

It will be noticed that *i* is the vertical mirror picture of *o* and *ro* of *ri*, and *vice versa*. Further if the mirror is placed upon one of the horizontal lines, *ri* is the mirror picture of *o* as well as *ro* of *i* and *vice versa*.

There are four more arrangements. There is the Chinese way of writing downward in vertical columns as well as its inversion, and the reversed order of both. This method originated by the use

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

1	63	62	4	5	59	58	8
56	10	11	53	52	14	15	49
48	18	19	45	44	22	23	41
25	39	38	28	29	35	34	32
33	31	30	36	37	27	26	40
24	42	43	21	20	46	47	17
16	50	51	13	12	54	55	9
57	7	6	60	61	3	2	64

1	143	142	4	5	139	138	8	9	135	134	12
132	14	15	129	128	18	19	125	124	22	23	121
120	26	27	117	116	30	31	113	112	34	35	109
37	107	106	40	41	103	102	44	45	98	95	48
49	95	94	52	53	91	90	56	57	87	86	60
84	62	63	81	80	66	67	77	76	70	71	73
72	74	75	69	68	78	79	65	64	82	83	61
85	59	58	88	89	55	54	92	93	51	50	96
97	47	46	100	101	43	42	104	105	39	38	108
36	110	111	33	32	114	115	29	28	118	119	25
24	122	123	21	20	126	127	17	16	130	131	13
133	11	10	136	137	7	6	140	141	3	2	144

EVEN SQUARES IN MULTIPLES OF FOUR.*

* These squares, 4×4 and its multiples, consist of *o* and *ro* orders only, and it will be sufficient to write out the two 4×4 squares, which show how *o* and *ro* are mutually interchangeable.

o	ro	ro	o
ro	o	o	ro
ro	o	o	ro
o	ro	ro	o

ro	o	o	ro
o	ro	ro	o
o	ro	ro	o
ro	o	o	ro

of bamboo strips as writing material in China, and we may utilize the two vowel sounds of the word "bamboo" (viz., *a* and *u*) to name the left and the right downward order, *a* the left and *u* the right, the reverse of the right *ru* and of the left *ra*; but for our present purpose there will be no occasion to use them.

Now we must bear in mind that magic squares originate from the ordinary and normal consecutive arrangement by such transpositions as will counteract the regular increase of value in the normally progressive series of figures; and these transpositions depend upon the location of the several cells. All transpositions in the cells of even magic squares are brought about by the substitution of figures of the *ro, i,* and *ri* order for the original figures of the ordinary or *o* order, and the symmetry which dominates these changes becomes apparent in the diagrams, which present at a glance the order to which each cell in a magic square belongs.

Numbers of the same order are grouped not unlike the Chladni acoustic figures, and it seems to me that the origin of the regularity of both the magic figures and this phenomenon in acoustics, is due to an analogous law of symmetry.

The dominance of one order *o, ro, i,* or *ri,* in each cell of an even magic square, is simply due to a definite method of their selection from the four different orders of counting. Never can a figure appear in a cell where it does not belong by right of some regular order, either *o, ro, i,* or *ri.*

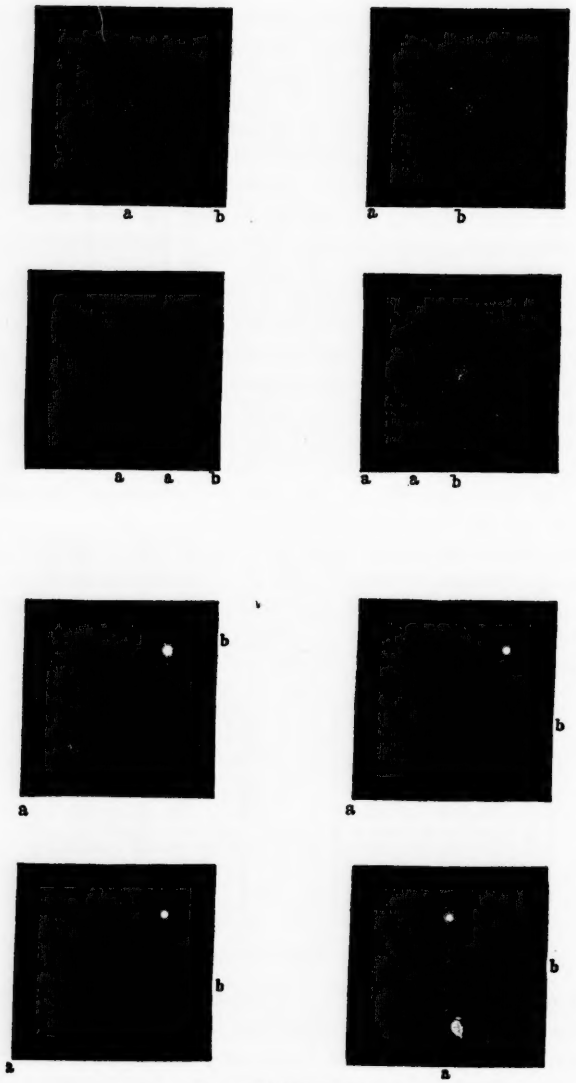
1	35	4	33	32	6
12	8	28	27	11	25
24	17	15	16	20	19
13	23	21	22	14	18
30	26	9	10	29	7
31	2	34	3	5	36

1	5	33	34	32	6
30	8	28	9	11	25
18	23	15	16	20	19
24	14	21	22	17	13
7	26	10	27	29	12
31	35	4	3	2	36

6×6 EVEN SQUARES.

The magic square of 4×4, consists only of *o* and *ro* figures, and the same rule applies to the simplest construction of even squares of multiples of four, such as 8×8, and 12×12.

There are several ways of constructing a magic square of 6×6. Our first sample consists of 12 *o*, 12 *ro*, 6 *ri*, and 6 *i* figures The 12 *o* hold the diagonal lines. The 12 *ro* go parallel with one of



CHLADNI FIGURES.*

* The letter *a* indicates where the surface is touched with a finger; while *b* marks the place where the bow strikes the glass plate. In the four upper

these diagonals, and stand in such positions that if the whole magic square were diagonally turned upon itself, they would exactly cover the 6 *i*, and 6 *ri* figures. And again the 6 *i* and 6 *ri* also hold toward each other places in the same way corresponding to one another; if the magic square were turned upon itself around the other diagonal, each *ri* figure would cover one of the *i* order.

If we compare the magic squares with the sand-covered glass plates which Chladni used and think of every cell as equally filled with the four figures that would fall upon it according to the normal sequence of *o, ro, i,* and *ri*; and further if we compare their change into a magic square to a musical note harmonizing whole rows into equal sums, we would find (if by some magic process the different values of the several figures would mechanically be turned up so as to be evenly balanced in rows) that they would present geometrically harmonious designs as much as the Chladni acoustic figures.

The progressive transformations of *o, ro, i,* and *ri,* by mirroring, are not unlike the air waves of notes in which *o* represents the crest of the wave, *ro* the trough, *i* and *ri* the nodes.

In placing the mirror at right angles progressively from *o* to *i*, from *i* to *ro,* from *ro* to *ri,* and from *ri* to *o,* we return to the beginning thus completing a whole sweep of the circle.* The reverse of *o* which is *ro* represents one-half turn, *i* and *ri* the first and third quarter in the whole circuit, and it is natural, therefore, that a symmetry-producing wave should produce a similar effect in the magic square to that of a note upon the sand of a Chladni glass plate.

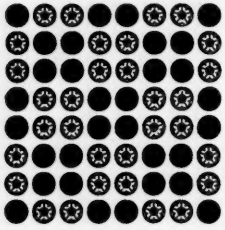
MAGIC SQUARES IN SYMBOLS.

The diagrams which are offered here, as samples, are the best evidence of their resemblance to the Chladni figures, both exhibiting in their formation, the effect of the law of symmetry. The most elegant way of rendering the different orders, *i, ri, o,* and *ro,* visible at a glance, would be by printing the cells in four different colors,

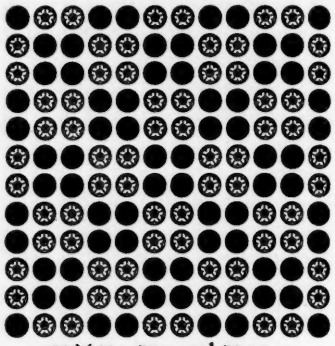
diagrams the plate has been fastened in the center, while in the lower ones it has been held tight in an excentric position, indicated by the white dot.

* See diagram on page 125.

but for proving our case, it will be sufficient to have the four orders represented by four symbols, omitting their figure values, and we here propose to indicate the order of *o* by ●, *ro* by ❁, *i* by ✢, *ri* by +.



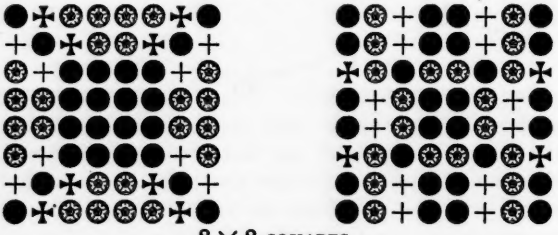
8 × 8. 32 *o* and 32 *ro*.



10 × 10. 72 *o* and 72 *ro*.

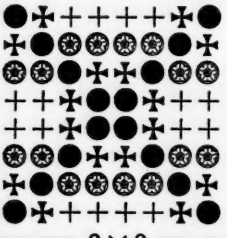
SQUARES OF MULTIPLES OF FOUR.

Constructed only of *o* and *ro*.



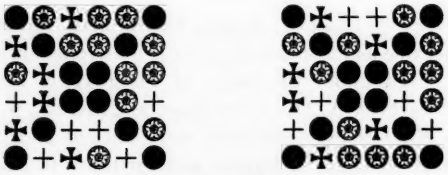
8 × 8 SQUARES.

Constructed from all the orders, *o*, *ro*, *i*, and *ri*.

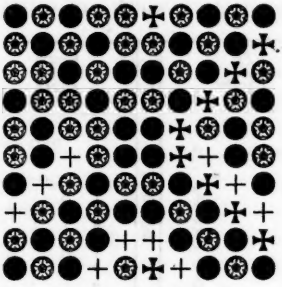


ANOTHER 8 × 8 SQUARE.

It will be noted that in this square the arrangement of the *o* symbols corresponds very closely to the distribution of the sand in the second of the Chladni diagrams. The same may be said of the two following figures, and it is especially true of the first one of the 8 × 8 squares just preceding.



12 *o*, 12 *ro*, 6 *i*, 6 *ri*.



40 *o*, 40 *ro*, 10 *i*, 10 *ri*.

The reader will notice that there is a remarkable resemblance between the symmetry displayed in this figure and in the fourth of the Chladni diagrams.

EXAMPLES OF 6×6 AND 10×10 MAGIC SQUARES.

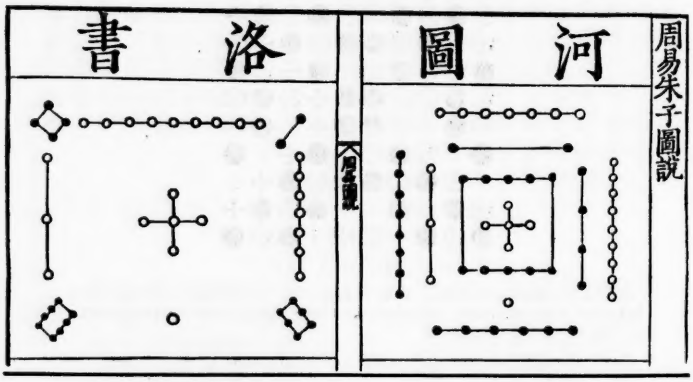
THE MAGIC SQUARE IN CHINA.

In the introduction to the Chou edition of the *Yih King*, we find some arithmetical diagrams and among them the *Loh-Shu*, the scroll of the river Loh, which is a mathematical square from 1 to 9, so written that all the odd numbers are expressed by white dots, i .e., yang symbols, the emblem of heaven, while the even numbers are in black dots, i. e., yin symbols, the emblem of earth. The invention of the scroll is attributed to Fuh-Hi, the mythical founder of Chinese civilization, who according to Chinese reports lived 2858-2738 B. C. But it goes without saying that we have to deal here with a reconstruction of an ancient document, and not with the document itself.

The first unequivocal apearance of the Loh-Shu in the form of a magic square is in the latter part of the posterior Chou dynasty (951-1126 A. D.) or the beginning of the Southern Sung dynasty (1127-1333 A. D.). The Loh-Shu is incorporated in the writings of Ts'ai Yüan-Ting who lived from 1135-1198 A. D. (cf. Mayers,

Chinese Reader's Manual, I, 754a), but similar arithmetical diagrams are traceable as reconstructions of primitive documents among scholars that lived under the reign of Sung Hwei-Tsung, which lasted from 1101-1125 A. D. (See Mayers, *C. R. M.*, p. 57.)

The *Yih King* is unquestionably very ancient and the symbols yang and yin as emblems of heaven and earth are inseparable from its contents. They existed at the time of Confucius (551-479 B. C.), for he wrote several chapters which are called appendices to the



THE SCROLL OF LOH. THE MAP OF HO.*

(According to Ts'ai Yüang-ting.)

TWO ARITHMETICAL DESIGNS OF ANCIENT CHINA.

Yih King, and in them he says (III, I, IX, 49-50. *S. B. E.,* XVI, p. 365.):

"To heaven belongs 1; to earth, 2; to heaven, 3; to earth, 4; to heaven, 5; to earth, 6; to heaven, 7; to earth, 8; to heaven, 9; to earth, 10.

"The numbers belonging to heaven are five, and those belonging to earth are five. The numbers of these two series correspond to each other, and each one has another that may be considered its mate. The heavenly numbers amount to 25, and the earthly to 30. The numbers of heaven and earth together amount to 55. It is

* The map of Ho properly does not belong here, but we let it stand because it helps to illustrate the spirit of the times when the scroll of Loh was composed in China. The map of Ho contains five groups of odd and even figures, the numbers of heaven and earth respectively. If the former are regarded as positive and the latter as negative, the difference of each group will uniformly yield $+5$ or -5.

by these that the changes and transformations are effected, and the spiritlike agencies kept in movement."

This passage was written about 500 B. C. and is approximately simultaneous with the philosophy of Pythagoras in the Occident, who declares number to be the essence of all things.

One thing is sure, that the magic square among the Chinese cannot have been derived from Europe. It is highly probable, however, that both countries received suggestions and a general impulse from India and perhaps ultimately from Babylonia. But the development of the yang and yin symbols in their numerical and occult significance can be traced back in China to a hoary antiquity so as to render it typically Chinese, and thus it seems strange that the same idea of the odd numbers as belonging to heaven and the even ones to earth appears in ancient Greece.

I owe the following communication to a personal letter from Professor David Eugene Smith of the Teachers' College of New York:

"There is a Latin aphorism, probably as old as Pythagoras, *Deus imparibus numeris gaudet.* Virgil paraphrases this as follows: *Numero deus impare gaudet.* (Ecl. viii, 75). In the edition I have at hand* there is a footnote which gives the ancient idea of the nature of odd and even numbers, saying:

"*...impar numerus immortalis, quia dividi integer non potest, par numerus mortalis, quia dividi potest; licet Varro dicat Pythagoreos putare imparem numerum habere finem, parem esse infinitum* [a curious idea which I have not seen elsewhere]; *ideo medendi causa multarumque rerum impares numeros servari: nam, ut supra dictum est, superi dii impari, inferi pari gaudent.*

"There are several references among the later commentators to the fact that the odd numbers are masculine, divine, heavenly, while the even one were feminine, mortal, earthly, but I cannot just at this writing place my hands upon them.

"As to the magic square, Professor Fujisawa, at the International Congress of Mathematicians at Paris in 1900, made the assertion that the mathematics derived at an early time from the Chinese (independent of their own native mathematics which was of a somewhat more scientific character), included the study of

* P. Virgilii Maronis | Opera, | cum integris commentariis | Servii, Philargyrii, Pierii, | Accedunt | Scaligeri et Lindenbrogii | . . . . . | Pancratius Masvicius | . . . | Tom. I, | . . . | Leonardiae, | . . . | . . cIↃIↃccxvii.|

these squares, going as far as the first 400 numbers. He did not, however, give the dates of these contributions, if indeed they are known."

As to other magic squares, Professor Smith writes in another letter:

"The magic square is found in a work by Abraham ben Ezra in the eleventh century. It is also found in Arabic works of the twelfth century. In 1904, Professor Schilling contributed to the Mathematical Society of Göttingen the fact that Professor Kielhorn had found a Jaina inscription of the twelfth or thirteenth century in the city of Khajuraho, India, a magic square of the notable peculiarity that each sub-square sums to 34."

Here is the square which Professor Smith encloses:

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

We must assume that we are confronted in many cases with an independent parallel development, but it appears that suggestions must have gone out over the whole world in most primitive times perhaps from Mesopotamia, the cradle of Babylonian civilization, or later from India, the center of a most brilliant development of scientific and religious thought.

How old the magic square in China may be, is difficult to say. It seems more than probable that its first appearance in the twelfth century is not the time of its invention, but rather the date of a recapitulation of former accomplishments, the exact date of which can no longer be determined.

THE JAINA SQUARE.

Prof. Kielhorn's Jaina square is not a "perfect magic square" according to Mr. Andrews' definition, quoted above. While the sums of all the rows, horizontal, vertical, and diagonal, are equal, the figures equidistant from the center are not equal to $n^2 + 1$, viz., the sum of the first and last numbers of the series. Yet it will be seen that in other respects this square is more perfect, for it represents

a distribution of the figure values in what might be called absolute equilibrium.

First we must observe that the Jaina square is continuous, by which I mean that it may vertically as well as horizontally be turned upon itself and the rule still holds good that wherever we may start four consecutive numbers in whatever direction, backward or forward, upward or downward, in horizontal, vertical, or slanting lines, always yield the same sum, viz. 34, which is $2(n^2+1)$; and so does any small square of 2×2 cells. Since we can not bend the square upon itself at once in two directions, we make the result visible in the adjoined diagram, by extending the square in each direction by half its own size.

10	5	16	3	10	5	16	3
15	4	9	6	15	4	9	6
1	14	7	12	1	14	7	12
8	11	2	13	8	11	2	13
10	5	16	3	10	5	16	3
15	4	9	6	15	4	9	6
1	14	7	12	1	14	7	12
8	11	2	13	8	11	2	13

THE SQUARE EXTENDED.

Wherever 4×4 cells are taken out from this extended square, we shall find them satisfying all the conditions of this peculiar kind of magic squares.

The construction of this ancient Jaina equilibrium-square requires another method than we have suggested for Mr. Andrews' "perfect squares," and the following considerations will afford us the key:

First we write the numbers down into the cells of the square in their consecutive order and call the four rows in one direction A, B, C, D; in the other direction 1, 2, 3, 4. Our aim is to redistribute them so as to have no two numbers of the same denomination in the same row. In other words, each row must contain one and only one of each of the four letters, and also one and only one of each of the four figures.

	1	2	3	4
A	1	2	3	4
B	5	6	7	8
C	9	10	11	12
D	13	14	15	16

In Consecutive Order.

A_1	B_4	C	D
D_2	C_3	B	A
3	2		
4	1		

The Start for a Redistribution.

A_1	B_4	C_2	D_3
D_2	C_3	B_1	A_4
B_3	A_2	D_4	C_1
C_4	D_1	A_3	B_2

The Perfected Redistribution.

1	8	10	15
14	11	5	4
7	2	16	9
12	13	3	6

Figure Values of the Square.

CONSTRUCTING THE SQUARE.

We start in the left upper corner and write down in the first horizontal row the letters A, B, C, and D, in their ordinary succession, and in the second horizontal row, the same letters in their inverted order. We do the same with the numbers in the first and second vertical rows. All that remains to be done is to fill out the rest in such a way as not to repeat either a letter or a number. In the first row there are still missing for C and D the numbers 2 and 3, of which 2 must belong to C, for C_3 appears already in the second row and 3 is left for D.

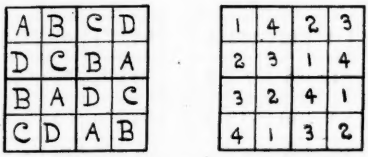
In the second row there are missing 1 and 4, of which 1 must belong to B, because we have B_4 in the first row.

In the first vertical row the letters B and C are missing, of which B must belong to 3, leaving C to 4.

In the second vertical row A and D are missing for 1 and 2. A_1 and D_2 exist, so A must go to 2, and D to 1.

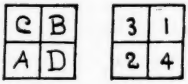
In the same simple fashion all the columns are filled out, and then the cell names replaced by their figure values, which yields the same kind of magic square as the one communicated by Prof. Smith, with these differences only, that ours starts in the left corner with number 1 and the vertical rows are exchanged with the horizontal ones. It is scarcely necessary to point out the beautiful symmetry in the distribution of the figures which becomes fully apparent when we consider their cell names. Both the letters, A,

B, C, D, and the figures, 1, 2, 3, 4, are harmoniously distributed over the whole square, so as to leave to each small square its distinct individuality, as appears from the following diagram:



THE ARRANGEMENT OF LETTERS AND FIGURES.

The center square in each case exhibits a cross relation, thus:



In a similar way each one of the four groups of four cells in each of the corners possesses an arrangement of its own which is symmetrically different from the others.

* * *

I have had some correspondence on the Jaina square with Mr. W. S. Andrews, and after a perusal of my article he sends me the following suggestions:

"There are some secondary features connected with magic squares apart from their chief characteristic that every main column sums up to the same total amount. Two of these secondary features are as follows:

"First—That any two numbers which are equidistant from the center of the square shall amount to the sum of the first and last numbers of the series.

"Second—That every sub-diagonal column shall sum up to the same total amount as the main columns, thus making a square which is 'continuous' in any direction from any column.

"The ordinary magic square of 4×4 possesses the first characteristic but not the second. The Jaina square possesses the second characteristic but not the first. The 5×5 magic square given in Fig. 5 of my article possesses both characteristics."

Mr. Andrews encloses the following diagrams, adding thereto a few explanations.

I.

A_1	D_3	D_2	A_4
C_4	B_2	B_3	C_1
B_4	C_2	C_3	B_1
D_1	A_3	A_2	D_4

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

ORDINARY 4×4 MAGIC SQUARE.

II.

B_3	C_4	A_1	D_2
A_2	D_1	B_4	C_3	
D_4	A_3	C_2	B_1
C_1	B_2	D_3	A_4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

THE JAINA SQUARE.

III.

A_1	B_4	C_2	D_3
D_2	C_3	B_1	A_4
B_3	A_2	D_4	C_1
C_4	D_1	A_3	B_2

1	8	10	15
14	11	5	4
7	2	16	9
12	13	3	6

THE JAINA SQUARE MODIFIED BY DR. CARUS.

IV.

B_5	D_3	A_1	C_4	E_2
C_1	E_4	B_2	D_5	A_3
D_2	A_5	C_3	E_1	B_4
E_3	B_1	D_4	A_2	C_5
A_4	C_2	E_5	B_3	D_1

10	18	1	14	22
11	24	7	20	3
17	5	13	21	9
23	6	19	2	15
4	12	25	8	16

5×5 MAGIC SQUARE.*

Referring to the small numbers in these squares it will be noticed that:

In square I,—The sum of any two numbers that are geometrically opposite to each other $= n + 1$.

In squares II and III,—The sum of the numbers in each column, *including the sub-diagonals,* is equal to $n(n+1)/2$.

* See Fig. 5 of Mr. Andrews' article on "Magic Squares," in the October *Monist,* page 432.

This feature implies *continuity* in the derived magic squares.

In square IV,—*Both* of the above characteristics obtain.

Mr. Andrews adds that he has not succeeded in making any arrangement of letters and numbers in squares I, II, or III which will embody *both* of the above characteristics.

THE PHILOSOPHICAL SIGNIFICANCE OF NUMBER.

Although magic squares have no immediate practical use, they have always exercised a great influence upon thinking people. It seems to me that they contain a lesson of great value in being a palpable instance of the symmetry of mathmatics, throwing thereby a clear light upon the order that pervades the universe wherever we turn, in the infinitesimally small interrelations of atoms as well as in the immeasurable domain of the starry heavens, an order which, although of a different kind and still more intricate, is also traceable in the development of organized life, and even in the complex domain of human action.

Magic squares are a visible instance of the intrinsic harmony of the laws of number, and we are thrilled with joy at beholding this evidence which reflects the glorious symmetry of the cosmic order.

Pythagoras says that number is the origin of all things, and certainly the law of number is the key that unlocks the secrets of the universe. But the law of number possesses an immanent order, which is at first sight mystifying, but on a more intimate acquaintance we easily understand it to be intrinsically necessary; and this law of number explains the wondrous consistency of the laws of nature. Magic squares are conspicuous instances of the intrinsic harmony of number illustrating the harmony of the spheres, and so they will serve as an interpreter of the cosmic order that dominates all existence.

Magic squares are a mere intellectual play that illustrates the nature of mathematics, and. incidentally, the nature of existence dominated by mathematical regularity. They illustrate the intrinsic harmony of mathematics as well as the intrinsic harmony of the laws of the cosmos.

The wisdom which we can trace in the cosmic order is not artificial, not the product of premeditated thought, not the result of an excogitated plan, but innate in existence itself. It is necessary

and absolute. It has not originated by the fiat of a God. It is without origin and without an end, being itself eternal and omnipresent; it is immutable, irrefragable, and unqualified, which means that it is without exception and its efficiency without conditions. It is uncaused, that is to say, it is not the effect of a cause, but is itself the principle of the law of causation.

Spinoza calls it "*causa sui*" (that is, cause of itself), but we take exception to this term as self-contradictory. There is no "cause of itself,"—that is, an effect that would be its own cause, nor is there an uncaused cause, a first cause. Cause and effect are always successive events; and every cause is the effect of a prior cause. Cause is some act or motion or disturbance of any kind, which, by a change in the system of conditions, calls forth further changes leading to a re-arrangement of the conditions, which is called the effect. What Spinoza means to say is that God or the absolute, which he calls "*substantia,*" is the universal *raison d'être* of all things, the final principle of explanation, and the supreme determinant.

Spinoza does not distinguish between two important ideas, *cause* and *reason,* and instead of speaking, as he intends to do, of the ultimate reason of the world, he speaks of the first cause, thus producing a confusion which down to the present day has proved a stumbling block to innumerable philosophers who succeeded him. Reason is the answer to the question *why.* Reason, unlike cause, which is always an event in the objective world, formulates a law that explains some event or class of events. Hence reason in the subjective sense can mean only our argument; and in the objective sense, the general law which dominates the course of events to be explained. All reasons intended to explain the isolated events of our experience, form a great system, and so comprehension consists practically in systematizing knowledge, and the different reasons can be subsumed under general laws, and can be represented as being finally reducible to one universal *raison d'être.*

All reasons considered by themselves, constructed independently of material existence, have been worked out in the sciences of pure form, (mathematics with all its branches, arithmetic, algebra, geometry, etc.; logic, and what Kant calls pure nature-science) which would comprise a critical and scientific view of causality, of the law of preservation of matter and energy, and kindred ideas.

And what is the reason that these so-called absolute branches

of knowledge play such an important part in the comprehension of the world?

The purely formal sciences possess the strange quality of being constructed by the human mind without utilizing external experiences; they are what Kant calls *"a priori,"* that is, they antedate experience; their validity is not based upon experience, but conditions it; they hold good before we begin to make experiences, and experience presupposes their validity. If they were not valid, experience would be impossible. While thus the formal sciences seem to be mind-made, they are after all the basic laws of objective existence, and the agreement between the basic laws of the mind and the highest laws of objective existence, does not teach us, as Kant says, that the order of the world is purely subjective, and transferred from the thinking mind upon the surrounding cosmos, from our own mental or ideal inside into the actual and real outside of space, from the subject to the object—a system which he calls "critical idealism"; it teaches us the intrinsic necessity of the universal laws governing any possible kind of existence; and we learn thereby that their basic principle may be condensed in the one word —consistency. It means that whatever originates, exists, or is created, (if there is any consistency at all) will according to conditions produce a definite kind of an order.

In arithmetic we create a universe of figures by the process of counting; in geometry we create another universe by drawing lines in the abstract field of imagination, laying down definite directions; in algebra we produce magnitudes of a still more abstract nature expressed by letters; and if we think of the material world as being created by a God, it must have been brought about by producing bodily forms. In all these cases the first step producing the general conditions in which we move, lays down the rule to which all further steps are subject, and so every one of these universes is dominated by a consistency, producing a wonderful symmetry, which in the cosmic world has been called by Pythagoras "the harmony of the spheres."

The intrinsic necessity of symmetry explains better than Kant's idealism why the mind-made laws of mathematics agree so closely with the laws of nature governing the material universe; both are applications of the same order underlying every consistent system, and we know very well that the mathematician creates, not in subjective arbitrariness, but after the intrinsic necessity of the first start from his position; and the objective realities, the laws of nature, which correspond to his theorems, are not nonentities, not

abstractions without meaning, but they are the dominating factors that shape all things in particular and the world in general. The system of these factors has not been made by the creator to suit his plans of creation, but it is the worthiest part of the creator himself. Humanly speaking we may call it the eternal constitution of the creator's mind. It alone marks the divinity of God, whatever we may understand by that name, and it alone is the Godhead of God.

THE GOD PROBLEM.

I have discussed the God problem before, but the subject is of such great importance and far-reaching consequence, that I will discuss it again in this connection, even at the danger of repeating myself.

There are two hostile parties in religion confronting each other. The one explains the mystery of the world order by the assumption of a personal God, an omnipotent, divine personality, in whose image man has been created; a being, who, like man, is possessed of will, intelligence and sentiment, who thinks and acts and plans as we do; the other party reduces everything to chance with its infinite possibilities. The two theories exclude one another, so that if one be true the other is necessarily wrong. Advocates of the former believe that God fashioned the world order to suit his purpose; those of the latter look upon the universe as a soulless, mindless, meaningless congeries of matter, exhibiting in its whirl of commotion many accidental uniformities. It is frequently assumed that no third proposition is possible, since these two appear to be contradictory. Because the iconoclastic proposition is very unsatisfactory, theists argue that we should accept belief in a personal God, on the ground that it is after all the most comforting and satisfactory solution. We, however, deem both positions to be faulty, and their contradictory feature is due solely to a slight misunderstanding of the true nature of the constitution of the world, as well as the significance of the God idea. Thus we take a middle ground which concedes to both parties some of their most vital contentions, and at the same time criticizes both for misstatements, which necessarily obscure the whole issue.

On the one side, we grant to the atheist that the world order can not have been fashioned by a God, for it is itself immutable and eternal; but at the same time we criticise in the atheist's posi-

tion his lack of appreciation of the world order as both the creative factor and the standard of moral conduct and truth; or, to speak religiously, as the divinity immanent in nature. We agree with the materialist that all concrete existence is material, we criticise him when he claims that there is no reality except that of matter. The interrelation of things, the formal aspect of being, and with them the laws of pure form, are not nonentities but the most essential features of reality. Neither matter nor energy is the most important part of the world. That which makes things such as they are is their form; and though the laws of form seem to be pure nothingness because they are not material entities, they are nevertheless the quintessence of being, for they alone, and not material things, condition the differences that constitute better and worse, higher and lower, valuable and worthless, good and evil, right and wrong, truth and falsehood. Stones and all other bodily objects are real; but the laws of nature, though not material, are after all actual presences in the world. They are more than real, they are superreal. Without them life would possess no significance and so we see that the materialist is onesided in his pretensions.

In extenuation of the errors of materialism we must remember that it was invented to deny the ghost existence of spirit. Spirit should not be regarded as a thing-in-itself. But neither should matter,—and as for that, spiritual life is as real as form, and the laws of form to which it owes its existence.

The theist appreciates the dignity of the cosmic order as a moral factor in the world, and is even right in representing it as the creator, the ruler, the supreme judge; but he is wrong in attributing to God human form, and the human constitution of individual existence. A personal God, (i. e., the deity conceived as an individual being) is a mere allegory, and the theist falls a prey to the pagan tendency of primitive man when he takes religious allegories in a literal sense and insists on the very points where the allegory fails.

While the method of representing God as a person is justifiable as an allegory, because the symbol itself is dignified and appropriate (man's personality is after all the highest type among all the creatures in the whole realm of creation), we must not forget that the very attributes of divinity exclude some of those features which are essential in the personality of man. The attributes of God are eternality, omnipresence, universality, omniscience, unqualified justice, spirituality, etc. Eternality excludes at once the assumption of God's thought being successive like the thinking of man. In

Isaiah (lv. 9) God says,—"As the heavens are higher than the earth, so are....my thoughts than your thoughts," and we insist that the difference betwen God's thought and man's is not even a mere difference of degree. While man's thoughts are temporal, being successive in time, God's are above time: they are simultaneous and omnipresent like the laws of nature. They are not dependent on time and space, but are operative in potent and immediate efficacy. Man's consciousness is the product of mental activity; it originates from feeling, and is practically nothing but an intensified state of sentiency. It is needed as a guide for man's life, so as to make it possible for him to adapt himself to circumstances. God is not in need of any adaptation to circumstances. He is not possessed of a consciousness like that of man, and the consistent conception of omniscience excludes the mechansim of consciousness which admits the several ideas, or objects of thought only successively within the limits of special attention. Omniscience means that all the divine thought is at once present in God. As eternality means that they are above time, so omniscience means that they are above space. They are not concentrated on a special spot after the fashion of cerebral activities in man, but they are independent of any mechanism. The materialist would say that because they are not material things such as stones or our own bodies, that they are non-existent; but we claim that for all that they are efficient; and if they are not real in the narrow sense of the word, if they are not concrete objects, they are nevertheless efficient presences, and so we rightly call them super-real.

The same is true as to the other divine attributes. The interadjustment of the whole harmony that pervades the cosmic order is so perfectly balanced that no fault can be found with it, and where we would take exception to the evils of existence, we will always find out, if we only patiently investigate, that the basic principle is right and could not be changed. Justice in the constitution of the world is nothing but the recognition of right adjustment, and so whatever the ills of life may be, God will remain justified.

While the iconoclasts, atheists, materialists, and kindred thinkers, look upon the uniformities of nature as accidental, we regard them as intrinsically necessary. According to our experience we find nothing in life which is not dominated by uniformity, and all events that seem to be isolated, and would not admit of being classified in the system of uniformities built up by science, are simply evidences of our ignorance, and though there be many of

them, there is no scientist who would look upon them in a different light. The whole world with all its phases, is subject to this immutable order of natural laws, and the assumption of accidental origin is absolutely excluded. The world order is a reality, or rather a superreality. It is possessed of all the divine attributes, and being a systematic whole, it constitutes a specific organism of a definite character, comparable to a personality.

While the cosmic order is not like man, we recognize at once that man was created in its likeness; for man originated by a recognition of universals, by discovering the general in the particular. The precedence of man over the beast is his reason, and man's reason (the *logos* of Neo-Platonism) is nothing but the becoming conscious of the uniformities of nature and interpreting them as universals, as instances of a type, and by comprehending the laws in which single cases find their application.

This idea has been indicated in the Gospel where it is said that the logos (that is the logical world order) was with God in the beginning and that it was God himself. Man, however, has become man only by a participation in the logos. To the extent that the logos has become incarnate in man, man has realized his humanity, and the highest ideal of manhood is the God-man, the logos incarnate.

Though God is not personal in the sense of human personality, we must not say that he is *im*personal, for that would appear as if divinity were less than man. The very opposite is the case. God being the prototype of man, man being formed in the likeness of God by incorporating in himself the divine logos, we must recognize that God, the prototype of all personality, is not less than a person, but more than a person, and noticing that the totality of divine thought which we trace in the laws of nature, constitutes a system not unlike the organism of a personal being, we prefer to characterize God as superpersonal. The negation of the human in God does not deprive him of anything divine, and the divine features which make man akin to God, are better, nobler and more divine than they are in any individual personality.

We do not limit God to the laws of nature, although we do not hesitate to characterize our God-conception as "Nomotheism." But by *nomos* we understand any norm, and God is the entirety of all that is eternally and universally normative. This eternal norm shows itself most palpably in the laws of reason and laws of nature, but we do not mean to limit our view of God to that feature of the

cosmic order which the naturalist formulates as natural laws, and the logicion as logical theorems. That aspect of God which is formulated by science is only a part of his being, and important though it may be, it is not the only important one. There are other aspects within the reach of our knowledge, and other additional possible ones. There is the aspect which the mystic takes in his devout attitude of vague feeling; there is the aspect of adoration in the worshiper; there is the aspect of aspiration to an ideal that glows in the soul of the moralist, and none of them is sufficient to exhaust the significance of the divine presence in which we live and move and have our being. No fault can be found with any attempt of the human mind to reach out in appreciation of the divine spirit, nor in any yearning for a closer and more intimate union with him, if only those who accept these several aspects remain conscious of their insufficiency, and would not, as does the dogmatist, take allegories literally and look upon their own individual and particular standpoint as the only possible and only allowable one.

I am far from saying that all aspects are of equal value: on the contrary, we can easily discriminate between the lower and the higher, between the more or less true, between the more or less matured, more or less free from superstition, more or less purified, more or less ennobling. I insist only on the insufficiency of any one of them which leaves out a recognition of the others.

God, although not a material being, although nowhere in space, and never in time, nowhere here or there, and never now and then, is not a nonentity. Though he is not a force, nor an energy, nor even the sum total of energy in the world, he is not inefficient. On the contrary he is omnipotent. His power is not to be measured in foot-pounds, yet his will, if we may use this humanizing term allegorically, is irresistible. He is an all-efficient presence in all happenings, and there is nothing that does not subsist through him.

Nomotheism is not pantheistic. Pantheism identifies God and the universe, and according to nomotheism, God is different from the concrete and material universe. God is in the universe, and it so happens that the old paradox is justified, that the part is greater than the whole. But God is not only present in the universe, he is present even in non-existence; that is to say the purely formal laws of being hold good not only for the actualized world, but for any possible world, for all the future, for all the past, for all the sys-

tems of thought constructed by science, and even for those universes that will never be created. The eternal norm of the divine principle is actualized in nature, but it is above nature, because nature is, as it were, only a single instance of the infinite possibilities to which it applies or could apply. Thus God is not limited to the cosmic order of nature, but is supernatural in the literal sense of the word. No system of things or beings, either real or imaginary, could exist without him, except perhaps a chaos; yet even that would be so filled with self-contradictory inconsistencies or irrational unthinkables that (if it be a true chaos) it would be thinkable only as a vague idea; it could not stand, for it would be doomed to die even before an attempt were made to call it into existence.

There is no prophet that preaches the superpersonal God more plainly than mathematics, and the magic squares are like a magic mirror which reflects a ray of the symmetry of the divine norm immanent in all things, in the immeasurable immensity of the cosmos not less than in the mysterious depths of the human mind.

EDITOR.

MR. PETERSON'S PROPOSED DISCUSSION.

Very valuable ideas ofttimes appear so obvious, when once set forth, that high laudation of their inventors would invite ridicule. Such, we are told, was the notion that obseded C. Colombo, and such is Mr. Peterson's proposal to start in *The Monist* a discussion of philosophical terminology. It may be a very simple proposal, but nobody, as far as one careful reader of *The Monist* remembers, had made it before; and its utility to students of phenomenology, normative science, and metaphysics will have a high co-efficient in its proportionality to the advantage they take of it. Duty calls upon us to contribute, each one what he can that will be useful, whether in the way of question or in that of answer. It seems likely that in my life-time of study I may have learned something of the way to investigate questions such as Mr. Peterson puts; and if so, here is an opportunity to be of aid to other students.

Experience, the first term concerning which Mr. Peterson asks for light, is somewhat remarkable for having been employed as nearly as possible in the same sense from Polus the Acragentine (i. e. native of Girgenti) sophist down to Avenarius and Haeckel.

As my first step in investigating its meaning, I should look out its equivalent *empeiria*,[1] in Bonitz's *Index Aristotelicus.* For every serious student of philosophy ought to be able to read the common dialect of Greek at sight, and needs on his shelves the Berlin Aristotle, in the fifth volume of which is that index. On looking out *empeiria* there, what first strikes one is that it is not a very common word with Aristotle, nor yet an unusual one, since Bonitz cites something over a dozen passages in which it occurs. The first (*Post. Anal.* II, xix) runs: "From sense are engendered memories, and from multiplied memory of the same thing is engendered experience; for many memories make up a single experience." Waitz (*Organon,* II, 429,) has a minute note on this passage. Another passage to which the Index refers (*Nic. Ethics,* VI, viii,) is thus translated by Stewart in his valuable "Notes" on the work: "If we ask why a boy may be a mathematician, but cannot understand philosophy or natural science, we find that it is because the truths of mathematics are abstract" [a bad explanation but that does not affect the evidence as to the meaning of *empeiria,*] "whereas the principles of philosophy and natural science are reached through long *experience.* A boy does not realize the meaning of the principles of philosophy and natural science, but merely repeats by rote the formulae used to express them." In the *Politics* (A, xi) Aristotle remarks that theorizing is free, while experience is necessitated, and goes on to speak of experience with live stock, etc. In another place in the *Politics* (E, ix) he says that the military commander of greatest experience in strategy is to be preferred, even though his habit of peculation be known; while for the chief of police, or for a treasurer, experience is of no account in comparison with integrity. But the cynosural passage is the first chapter of Book A of the *Metaphysics;* and here he remarks (as he likewise does in the *Ethics,*) that experience is a knowledge (*gnosis*)[2] of singulars. Therein Aristotle's language differs from that of the Socrates of Plato, with whom *empeiria* is the skill that results from long dealings with any matter. Aristotle never intended to say that there is no other cognition of singulars than in experience; for that would directly contradict his doctrine that experience is a mass of memories relating to the same subject. His remark was, however, understood in the Middle Ages to be a *definition* of experience, and was repeated as such, a blunder that was not so unnatural as it would have been if the scholastic doctors had dealt

[1] ἐμπειρία. [2] γνῶσις.

with direct experience. The teachings of the *Aristotelic Index* having been exhausted, I turn to Harper's *Latin Lexicon,* which informs me that no writer of the Golden Age used *experientia* in the general sense, though that acception became common in the Silver Age, especially with Tacitus. The next work that I personally should consult would be my own notes collected during more than forty years. I always carry a pad of the size of a Post Card, of thick papers, (50 in a pad, enough to last for two days, at least); and on these I note whatever elements of experience may reach me. I keep these in drawers and boxes like the card catalogue of a library. I arrange and rearrange them from time to time. It is a treasure more valuable than a policy of insurance. I probably have near two hundred thousand such notes. But in order to bring what I have to say to a close, I will quote from the definition of experience given by the father of modern experiential philosophy, Dr. John Locke. In the *Essay concerning Humane*[3] *Understanding,* (II, i, 2) we read (and the italics are in the original): "Whence has [the mind] all the materials of reason and knowledge? To this I answer, in one word, from *experience*: in that our knowledge is founded, and from that it ultimately derives itself. Our observation employed either about *external sensible objects, or about the internal operations of our minds, perceived and reflected on by ourselves, is that which supplies our understanding with materials of thinking.*" This definition so formally stated, by such an authority, quite peerless for our present purpose, should be accepted as definite and as a landmark that it would be a crime to displace or disturb. For in order that philosophy should become a successful science, it must, like biology, have its own vocabulary; and as in biology, it must be the rule that whoever wishes to introduce a new concept is to invent a new word to express it. This is no suggestion of the moment. I am, for my humble part, maturely convinced that philosophy will never be upon the road to sound results until we dismiss our affection for old words and our dislike of newfangled words, and make its vocabulary over after the fashion of taxonomic zoölogy and botany. I limit my recommendation to technical terms; for I can pretend to no competence to give advice about *belles-lettres.* Yet even there I perceive that people read old authors, and admire them for saying what they never meant to say; because the modern readers forget that two or three centuries ago words still familiar

[3] *Humane* and *human* were one and the same word in Locke's day.

suggested quite different ideas from those the same words now suggest.

But somebody may object that Locke's definition is vague, being founded on a misconception of the nature of perception. Suppose, the objector will say, that a newborn male infant were to be brought up among a colony of men on a desert island, without ever having seen a woman and barely having heard of such a creature. Suppose that, arrived at the age of twenty, he were to meet on the beach a Pacific Island woman who had swum over from another island. Would not the irresistible, the only possible cognition he could have of this creature be strongly colored by his own instincts? It would be the ineluctable result of "*observation employed concerning an external sensible object.*" The word "experience," however, is employed by Locke chiefly to enable him to say that human cognitions are inscribed by the individual's life-history upon a *tabula rasa,* and are not, like those of the lower animals, gifts of inborn instinct. His definition is vague for the reason that he never realized how important the innate element of our directest perceptions really is.

To such an objector I might say, My dear fellow, you must be joking; for under the guise of an objection you reinforce what I was saying with a new argument for restricting the use of the word "experience" to the expression of that vague idea which Locke so well defines. You make it plain that a distinct word is wanted, or rather two distinct words, to express the two more precise concepts which you suggest. The idea of the word "experience," was to refer to that which is forced upon a man's recognition, will-he nill-he, and shapes his thoughts to something quite different from what they naturally would have been. But the philosophers of experience, like many of other schools, forget to how great a degree it is true that the universe is all of a piece, and that we are all of us natural products, naturally partaking of the characteristics that are found everywhere through nature. It is in some measure nonsensical to talk of a man's nature as opposed to what perceptions force him to think. True, man continually finds himself resisted, both in his active desires and in that passive inertia of thought which causes any new phenomenon to give him a shock of surprise. You may think of an element of knowledge which thus resists his superficial tendencies; but to express precisely that idea you must have a new word: it will not answer the purpose to call it *experience.* You may also reflect that every man's environment

is in some measure unfavorable to his development; and so far as this affects his cognitive development, you have there an element that is opposed to the man's nature. But surely the word *experience* would be ill-chosen to express that.

But I am encroaching far too much upon the space of this number, and am taking too much advantage of our good editor's indulgence. I did wish to consider what element of his philosophy Comte had specially in mind in christening it *Positive*. He plainly meant that it should be unlike the metaphysical thought which kneads over and over what we know already, and would be like the sort of material which is furnished by a microscope or by an archæologist's spade. I hope Mr. Peterson's suggestion may bring a whole crop of fruit.

CHARLES SANTIAGO SANDERS PEIRCE.

MILFORD, PA.

BOOK REVIEWS AND NOTES.

WIE ICH WURDE WAS ICH WARD. Von *Julius Bahnsen.* Munich and Leipsic: Müller. 1905. Pp. lxxvii, 270.

Rudolf Louis, a friend and admirer of Julius Bahnsen, has edited an autobiography of this singular man, and accompanies it with an introduction in which are given the necessary data for an explanation of the author's significance. The book, however, is important mainly as a character study of a martyr of Prussian bureaucracy.

The editor, Rudolf Louis, calls Julius Bahnsen a philosopher, and we are far from grudging him this honor, if it may be considered as such, but the title is misleading, for the reader would expect a man of the type of Kant, Hegel or Spencer, or some one who has contributed toward the solution of philosophical or metaphysical problems. Bahnsen has done nothing of the kind. His contributions to philosophy are absolutely nil, but he is a philosopher in a broader sense. He is a thinker, a man of a contemplative disposition, who has pondered on his fate as well as his aspirations and surroundings. He was burning with ambition to do something for mankind, to be helpful to his fellow beings, especially his compatriots, and to have his ideals realized, but he has signally failed in this. Hence his life became a monotonous tragedy of a man forgotten, misjudged and ignored.

Rudolf Louis would lay the blame on the university professors who have silenced him, and thus cut off the arteries of his mental life; but the truth is that the university professors are quite innocent of his ill star, yet while it it appears that the main fault was inherent in the character of the man himself, we would not say that the Prussian bureacracy is free from blame. On the contrary, they should have been kinder toward him, and not have allowed an ingenious and aspiring man to wither in loneliness in a small town where he was deprived of the quickening spirit of sympathetic and congenial surroundings that would have helped to mature and educate this noble pessimist. Julius Bahnsen was an unfortunate character and his unhappy fate was predetermined by his own idiosyncrasies.

He was born in 1830 in Tondern in Schleswig, as the son of the principal of a teachers' seminary. He received the usual education, and when his little nation rebelled against the Danish yoke, he joined the Schleswig-Holstein company as a volunteer in 1849, and took part in its disastrous campaign, which ended in a lost battle and a shameful surrender. The participants, although formerly encouraged by Germany, were now exiled in 1851, and Bahnsen went to Tübingen to continue his studies as a philologist. At Tübingen

he took his degree as Doctor of Philosophy, but he did not feel at home among the Swabians. Though as a non-Prussian and an exile of Schleswig, he had the sympathy of his South German brethren, he felt the difference between the North and South German types too much not to look upon the Swabians as odd and uncongenial. In 1853 he accepted a position as tutor in Eutin-Schwartau; in 1854 he traveled to London; in 1855 to 1857 he taught in the private school at Altona. His favorite wish of settling in Oldenburg was not fulfilled. He applied for a position in the Gymnasium, but by one vote failed to be elected, and in 1858 he accepted an appointment as teacher at the Gymnasium at Anclam, Pomerania. If he found the Swabians uncongenial, he was more disappointed still in the Pomeranians, and he disliked especially the Prussian discipline, and the robust crudity of his Pomeranian colleagues as well as pupils. Under these circumstances he developed an anti-Prussian spirit which sought the salvation of Germany in a united empire, in which, however, every particular tribe should develop its own character and should be allowed to be itself.

His antipathy became more and more bitter and he was not a character to conceal his views, and so it was natural that both his colleagues and superiors were not favorably impressed with the Schleswig exile, and none of them tried to make him feel at home in his adopted country.

We must mention that Bahnsen had previously visited Schopenhauer mainly because his interest had been aroused by an unfavorable criticism of Schopenhauer's philosophy. We learn little of his interview with the great pessimist, and it seems that Schopenhauer's theoretical philosophy left no impression on Bahnsen, but the pessimism of the Frankfort philosopher agreed but too well with the misanthropy of the Schleswig exile, and his vain attempts to educate his pupils up to his own ideals, made him feel fully convinced that Schopenhauer's ideas of the unchangeableness of character were correct. His only work so far as we know in a properly philosophical or metaphysical line is an attempt at reconciling Schopenhauer's theory of the will with Hegel's dialectics. Not being known in university circles, however, his labors in this line were ignored while the author in his isolated position was not even afforded the salutary effect of friction by criticism.

As an educator and an observer of his surroundings Bahnsen concentrated his mind on a study of character, and published a series of essays on the subject under the title "Contributions to Characterology," which attracted some attention. But while his ability as an observer and thinker was recognized by his superiors, especially the ministerial counsellor, Ludwig Wiese, his sympathies with Schopenhauer could only make an unfavorable impression and branded him as an impracticable educator. Had he not himself acknowledged the impossibility of changing the character of his pupils by education? The result was that the counsellor after nine months wrote him a courteous letter, the real meaning of which, however, was written between the lines, which Bahnsen to his own disaster failed to understand. The counsellor expressed his view that he would find another sphere of life that would be more suitable for his talents. Bahnsen adds that it was in the power of the counsellor to afford him a more suitable occupation. He did not understand that a Prussian official was appointed to preserve the Prussian spirit of edu-

cation, and that the letter had gently hinted that Bahnsen might look for some position outside of Prussian educational institutions.

The opinion which the Prussian counsellor had formed of Bahnsen sealed his fate. It was transmitted to the authorities at Pomerania, and they executed the orders of the minister by transferring this dangerous man, who ventured to have views of his own in strict contradiction to Prussian notions, to a small town in a remote corner of farther Pomerania, where he received an appointment in a township gymnasium, depending entirely for his promotion on the will of a board of directors elected from the good citizens of the place. His promotion was not forthcoming, and his salary also remained small. An increase of salary had been promised him but it was not given, and when he appealed to the authorities they contended that they could not interfere with the local self-government of the place.

Bahnsen's lack of support on the part of the government was intentional. In fact we know from sources not contained in the book, and not known to Bahnsen and his friends, that they would have gladly discharged Bahnsen had they not preferred to avoid making a martyr of him, but to dwarf his spiritual growth and let his ambition die piecemeal of intellectual starvation. They have succeeded, and in reading over this unfortunate man's biography we ask the question whether he would have met with a better fate in life if he had left his uncongenial surroundings and sought another home in some other country. This is possible, but doubtful, for misanthropy was too integral a part of Bahnsen's character, and he was too proud to adapt himself to his surroundings.

As to his life at Lauenburg little remains to be said. He had married in 1863 Minnita Möller of Hamburg, but he lost her soon after the death of a baby girl in 1863. In Lauenburg he married the second time, Clara Hertzog, by whom he had four children, but this marriage was very unhappy and led finally to a divorce in 1874. No other reason was mentioned than incompatibility. In 1872 Bahnsen became acquainted with Hartmann, for whom he did some literary work, but while Bahnsen preserved his respect for Schopenhauer, he expressed a disdain for the philosopher of the unconscious, although in his enthusiasm he had called his third child Arthur Eduard Hartmann Bahnsen.

In Lauenburg he wrote different essays on "Will and Motive," 1870, (Stolp und Lauenburg i. P., H. Eschenhagen); "A Critical Discussion of the Hegel-Hartmann Evolutionism, Judged from the Standpoint of Schopenhauer," 1872, (Berlin: Carl Duncker); "A Contribution to the Philosophy of Language," 1877, (Leipsic: Wilhelm Wigand); "Mosaics and Silhouettes," 1877, (Leipsic: Otto Wigand); "The Tragic as the Law of the World and Humor as the Esthetic Form of Metaphysics," 1877, (Lauenburg, i. P.: F. Ferley); "Philosophy and Nationality," 1878, (Lauenburg i. P.: F. Ferley); "A Pessimist's Breviary, by an Initiate," 1879, (Berlin: Theobald Grieben), the first volume was published anonymously in 1879 and a second volume posthumously in 1882; "The Contradictions in Our Knowledge and in the Essence of the World," 1880, (Leipsic: Theobald Grieben); "Aphorisms of the Philosophy of Language, from the Standpoint of the Metaphysics of the Will," 1881, (Berlin: Theobald Grieben).

Bahnsen died on December 7, 1881, in his fifty-second year.

The title of the book is difficult to translate because the two words *wurde* and *ward* mean exactly the same thing, "became," and so are more than tautological, they are identical. It is characteristic of the odd idiosyncracy of the man. The publishers have added Bahnsen's picture as a frontispiece, which helps to explain his unique personality.

THE MESSIANIC HOPE IN THE NEW TESTAMENT. By *Shailer Mathews.* Chicago: University Press. Price, $2.50.

There is no doubt that Prof. Shailer Mathews of the University of Chicago is one of the ablest representatives of the school of higher Biblical criticism in this country. His recent work on *The Messianic Hope in the New Testament* is a valuable addition to theological literature along exegetical lines. The appearance of such thoughtful works as this written on a scientific historical basis by men of undoubted orthodoxy is the most hopeful indication of the time when the entire Christian Church will be purified of the primitive superstitious elements common to all nations in the childhood of their development, and will retain only such noble and inspiring truths as are in perfect accord with the broadening revelations of science; when it will recognize the kinship in thought of intelligent minds everywhere and of every affiliation. Special encouragement lies in the fact that this purification is coming from within the Church itself and that such scientific methods as this book represents not only are made use of by leaders of theological thought but are commended for practice to the many young students who in their turn are to influence the thought of a later generation.

The introductory chapter not only carries out its ostensible purpose of explaining the historical exegetical process which the author advocates for theological reconstruction, but also makes clear how far progressive Christian thinkers of to-day have advanced beyond a literal acceptance of the Scriptures. This deviation from the traditional interpretation is more general than we often realize. In speaking of the importance of separating the setting in which a thought is cast from its teaching, because people to-day do not think nor express themselves as they did in bygone centuries, Professor Mathews says: "How generally recognized this view has become in practical teaching may be seen in the abandonment of some of the most explicit directions of the New Testament on the ground that they were intended primarily and exclusively for Christians in some city like Corinth. Thus, for instance, few teachers would to-day assert that women should not speak in meetings, or that there was any divine regulation concerning the length of a Christian's hair. At the same time, these same teachers would assert that the general principles of orderly conduct and modest deportment which found expression in the apostle's directions to Græco-Roman Christians are as applicable to the Christians of to-day as to those of nineteen hundred years ago."

While thus repudiating the necessity of a word for word acceptance of the Bible, he believes the truth contained in it to be the object of revelation for he speaks of "criteria which shall enable one to distinguish the concepts and processes which conditioned the Biblical writers from the religious experience and truth which admittedly constitute the real substance of what we call revelation." These criteria are found in the popular concepts of Biblical times which, however, were not considered purely formal by those

who used them, but were doubtless intended to embody the truth as well as to typify it.

Professor Mathews treats in detail of the historical method which must be used to gain correct values in Biblical interpretation. The distinction must be kept in mind between the content of a teaching and the form in which it is cast, for men to-day cannot feel more than an "antiquarian interest" in anything but the content, and theologians of all men cannot afford to be anachronistic. The writer agrees with Professor Mathews that if his process be carried out to its consistent end there is nothing to be more desired in the direction of a scientific valuation of the foundations of the Christian faith, for "especially if the concept in question be one that obviously is derived from a cosmogony or a theology that does not square with historical and scientific facts and processes, it will not be difficult to give it its true value and signficance for the constructive and systematizing processes." It is not quite clear, however, that Profesor Mathews applies this criterion carefully to all of the leading concepts involved in the tenets of Christianity; for instance, the immaculate conception, resurrection, and special revelation as a source of knowledge for matters pertaining to the religious life.

In regard to the concept of messianism the historical process would consist in discovering what the concept actually was by studying its history in Judaism and then whether or to what extent it exists in the New Testament, in Jesus' own interpretation of his career, in its interpretation in the minds of the apostles and by the early Church. Messianism is the most obviously local and ethnic concept that influences the Gospel records, as it is the most frequent and consequently inevitable. Its importance for Christian theologians lies in the fact that it was "the medium through which his followers looked at Jesus, the form in which they expressed their appreciation of him, and the warp of all their speculation as to his and their own future."

The introduction closes with an outline of the course the author is about to pursue in discovering the part this concept played in the doctrines of the early Church, and in how far it is formal and how far essential Christianity:

"We shall first of all attempt to discover and formulate the elements of eschatological messianism as it is found in the literature of Judaism; in the second place, we shall examine the New Testament to see how much or how little of this element is to be found on its pages; and, in the third place, we shall attempt to determine the influence of such an element in Christian thought, and as far as possible to discover what would be the result upon historical Christianity if it were removed or, more properly speaking, allowed for."

The result of his investigation concerning the early Church is that "formally, the Church was a group of messianists awaiting a kingdom that never came and indifferent to all customs of society except those that were evil; essentially the Church was a group of men and women endeavoring to let the new religious and ethical life that had come to them from God through accepting Jesus as Christ express itself in social relations."

In general he concludes that there is an essential identity between the messianic hope of the two Testaments, the New modifying the Old when compelled to by actual facts in the life of Jesus. To allow for this concept of messianism, Professor Mathews believes, does not destroy the essentials of

Gospel teaching, but makes them the more intelligible and convincing. These essentials he enumerates as the personality, the teaching and the resurrection of Jesus; a rational faith in God as Father; a certainty of divine forgiveness; an experience of the eternal life; an assurance of a complete life beyond and because of death. He says, referring to the ideal Christian, "To make these facts dynamic in reason and will, he may use whatever world-view he may regard as the modern equivalent of messianism, or whatever terms he may regard as supreme definition of that divine Personality whom the first Jewish believers called the Messiah." L. G. R.

LA MORALE DANS LE DRAME, L'ÉPOPÉE ET LE ROMAN. Par *Lucien Arréat.* Paris: Alcan, 1906.

A third edition of M. Arréat's *Ethics in the Drama, the Epic and the Novel* has appeared at Alcan's. As it has been mentioned formerly in *The Monist* it will be sufficient now to recall its predominating thought which is the consideration of the dramatic actions imagined by poets as fictitious experiences which reproduce and interpret those of real life; and the use of the study of poetic creations as a means of criticising the moral systems which philosophers have built up as well as for the history of morality itself throughout the centuries and in social life.

Dramatic actions invented by poets are fictitious experiences, repeating and interpreting those of real life.

Systems of ethics are likewise interpretations. But the ideas which are matters for speculation to philosophical schools are lived out in novels and on the stage. The drama participates at the same time in both worldly and philosophical life. Therefore its creations seem specially adapted for illuminating the criticisms of systems established by philosophers, as well as to illustrate the formation and development of morality itself. Upon this idea is laid the foundation of M. Arréat's work.

In his preface he says: "If we were to investigate more closely the relation existing between the ethics of philosophers and of every-day life, between theoretical and practical ethics, we might say that the hero of a philosophy is an abstraction only to be found in books, and that the living individual of flesh and bone never personifies any doctrine, or if he does it is only incidentally. Neither the pure stoic, nor the pure Christian, nor the pure Kantian, nor the cold utilitarian actually exists. They are types which are either invented or proposed by our theories, but never figures taken from round about us. No ideal discipline takes hold of the whole man.

"Neither is the character in the drama this abstract hero, but he comes very near it, while the real living individual remains his model. The poet, however unconscious or indifferent he may be, carves his figures in a desired relief and chooses the circumstances which will allow him to design them with a precise outline. He borrows from history or legend such adventures as have moved the world or thrilled the imagination of men, or else he invents an action which he throws on the canvas of ordinary life. In any case he disposes facts with the purpose of throwing into relief a character which interests him, or a solution which satisfies his judgment. Belonging to his own time, the poet is imbued with the opinions that govern the age, and consequently gives a certain meaning to the theories he puts into prac-

tice. In creating he makes a moral experiment, and is permitted to conclude it as he chooses. It is enough that the action should be well managed, the experiment artistically worked out. The admiration of succeeding ages decides, and is given to work and literatures of widely differing genius."

SOCIOLOGICAL PAPERS. By *Francis Galton, E Westermarck, P. Geddes, E. Durkheim, Harold H. Mann* and *V. V. Branford.* With an Introductory Address by *James Bryce.* Published for the Sociological Society. London: Macmillan & Co. 1905. Pp. xviii, 292.

This volume consists of the papers read before the Sociological Society in its first session in 1904, together with a report of some of the discussions which followed them. The president's opening address, by Mr. James Bryce, fitly serves for an introduction. The history and methodology of sociology are represented by the papers of Mr. Branford and Professor Durkheim on the relation of sociology to the social sciences and to philosophy. Pioneer researches in borderland problems are represented by Dr. Westermarck's paper on the position of woman in early civilization, and that of Mr. Mann on agricultural village life in England. Applied sociology is represented by the paper of Professor Geddes on "Civics," and that of Mr. Galton on "Eugenics," in which the first treats of cities and the other of their citizens. In the preface, responsibility for statement of facts and opinions advocated, is disclaimed for the society and laid entirely upon the individual writers and speakers.

The fifth and sixth volumes of the Harvard Oriental Series are devoted to the Brhad-devata, which is attributed to Saunaka, and comprises a summary of the deities and myths of the Rig-veda. It is translated into English and critically edited by Arthur Anthony MacDonell, Boden Professor of Sanskrit in the University of Oxford. (Price, $1.50 per vol.)

The first volume contains the editor's introduction and the Sanskrit text together with seven indices of the Brhad-devata; while the second consists of the translation, each translated verse being immediately followed by its corresponding notes.

The Brhad-devata itself begins with a long introduction, consisting mainly of a classification and enumeration of the Vedic deities in connection with Prajapati and with the Vedic triad of Fire, Indra or Vayu, and Sun, as representing respectively the gods of the three worlds. This introduction closes with a grammatical discussion dealing with the different parts of speech, word-analysis, and a criticism of Yaska's errors in the division of words. Most of the work is concerned with stating the deities in right succesion for the hymns and stanzas of the Rig-veda. It is however by no means merely an index of the deities, but contains also forty legends whose purpose is to explain the occasion of the composition of the hymns with which they are connected. This is the oldest systematic collection of legends which we possess in Sanskrit. They are told in epic style, and are connected historically with many of the Mahabharata stories. The text contains many other data connected with the deities, and a full discussion of the character of the Vaisvadeva hymns.

One of the most interesting and original investigations of the much-mooted subject of personal immortality is Prof. Gustav Theodor Fechner's little book, *On Life After Death.* The English translation by Hugo Wernekke, Head Master of the Realgymnasium at Weimar, first appeared in 1882, and was presented to its venerable author on his eightieth birthday. The book is written in a very popular style, and since the edition has long since been exhauseted, the translator has thoroughly revised his work, which The Open Court Publishing Company now deems advisable to re-publish. A discussion of the publisher's position in regard to Professor Fechner's views, is contained elsewhere in this number of *The Monist.*

A CORRECTION.

To the Editor of The Monist:

In a discussion of the contemporary development of diplomacy in the July number of *The Monist* (Vol. XV, p. 474) the reviewer mentions as "the principle formulated by French kings in the impudent dictum, the ultimate reason of autocracy: *Car tel est notre bon plaisir*" and I regret to find here this trite assertion which is also a gross mistake.

Mr. Funck Brentano, who knows our seventeenth century so thoroughly, alluded to it quite recently in the review *Minerva* with reference to an historical novel by M. Henri De Régnier. "The *bon plaisir,*" writes he, "is diplomatically a heresy, a barbarism, as Count Mas-Latria demonstrated in such a scholarly way in his conclusive article published by *La Bibliothèque des Chartes.* The only formula used in the royal acts is *tel est notre plaisir,* and these words are handed down by tradition from the very distant times when *plaisir* had a far more serious meaning. "At present," writes Froissart, "he (a count of Flanders) is pleased to obey the ordinances and pleasures of the king of France and the French."

The pleasure, or "good pleasure," if the latter be preferred, was the king's verdict pronounced by him as head of the State, after due deliberation with his council, and which could be delivered only after it was countersigned by a secretary of the government. Many facts also show that *bon plaisir* was not more exact in a figurative than in a literal sense.

I thought it of some use to send you this correction for the information of your readers. I have no doubt but you will be glad to be warned against ready made judgments, repeated by partially or ill-informed people.

PARIS, FRANCE. LUCIEN ARRÉAT.

With the beginning of a new volume, we have decided to change the style of *The Monist* by having the more important essays printed in pica type, which we hope will make the reading of weighty matter easier; and we are especially glad to have secured for this first number of the new year, two articles on the physical constitution of the universe, a subject which is interesting to all speculative minds, and should therefore not be forgotten in philosophy. Both are popular in style so as to give a clear insight iito the present status of our knowledge as to the elementary composition of the smallest units of the materials that constitute the universe.

Professor Lindemann's speech on "The Form of Atoms" is well supplemented by Mr. W. S. Andrews' "Manifestations of the Ether."

Professor Lindemann, who happens to be Rector Magnificus of the University of Munich, has kindly sent us the German original of the present essay which has served him as the official speech of his rectorate, and for interested readers we will state that it has just appeared in the *Süddeutsche Monatshefte* (Verlag von Adolf Bonz & Co., Stuttgart) from which it can be had in separate pamphlet form. Reference to others of Professor Lindemann's more technical expositions of the subject will be found in a footnote under the text.

The article by Mr. W. S. Andrews has been specially solicited to provide our readers with a popular sketch of the physical nature of the universe, giving particular consideration to the significance of Röntgen rays, radium, etc. And we have succeeded in procuring the man who in our opinion is best fitted for this purpose. Mr. Andrews has devoted his life to this specialty, and has been in personal contact with Mr, Thomas A. Edison, as an assistant in his inventive work.

He entered the service of Mr. Edison in 1879 when the great inventor was commencing the experimental development of his incandescent electric lamp and all the accessory apparatus required for a complete system of general distribution of electricity for lighting and other industrial purposes. Mr. Andrews was peculiarly fitted for this work by inclination and training, and in 1881 he was placed in charge of the testing of all the electric generators and other apparatus then being made under the Edison patents. In 1883 Mr. Edison entrusted to him the supervision of the electrical equipment of the Edison Electric Lighting Stations, and in 1885 he was appointed chief electrical engineer of the Edison Co. for isolated lighting. Since that date he has occupied many responsible and important positions, devoting his energies chiefly to engineering work in the field of electric lighting.

During the past eight or nine years Mr. Andrews has devoted much of his spare time to the study of X rays, radio-activity and other physical phenomena of this character, and being benefited by the suggestions and advice of one of the greatest inventors that has ever lived, and following the natural bent of his own gifted mind, he has established a reputation for himself as an authority in these lines of modern investigation and discovery